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Preliminary analysis of gas release and dispersion behaviour relevant to the use of hydrogen in the natural gas distribution network

Presented by

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Health and Safety Executive

HSE

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RESEARCH AND GUIDANCE FROM HSE

Preliminary analysis of gas release and dispersion behaviour relevant to the use of hydrogen in the natural gas distribution network

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Research - HSE funded to provide evidence which underpins its policy and regulatory activities
Guidance - freely available to help people comply with health and safety law

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Outline

- Background
- Aims
- Gas properties
- Release rates
- Jet and plume dispersion
- Comparison to Quadvent
- Gas accumulation
- IGE/UP/1
- Conclusion

Background

- Climate change: UK to bring all greenhouse gas emissions to net zero by 2050 (Rt Hon Chris Skidmore, BEIS, 27 June 2019)
- Several ongoing projects investigating if it is technically feasible and safe to replace natural gas with hydrogen in the gas network, in commercial/residential buildings and gas appliances
  - HyDeploy: 20% hydrogen in natural gas } Existing gas network
  - H21: 100% hydrogen } Existing gas network
  - H100: 100% hydrogen in a new gas network
  - Hy4Heat: hydrogen appliances, gas quality criteria, meters

Aims

To address the following questions:

- Does hydrogen leak more than natural gas?
  - If so, by how much?
- What is its effect on the size of the flammable cloud?
- What are the implications for gas industry procedures, like IGE/UP/1?

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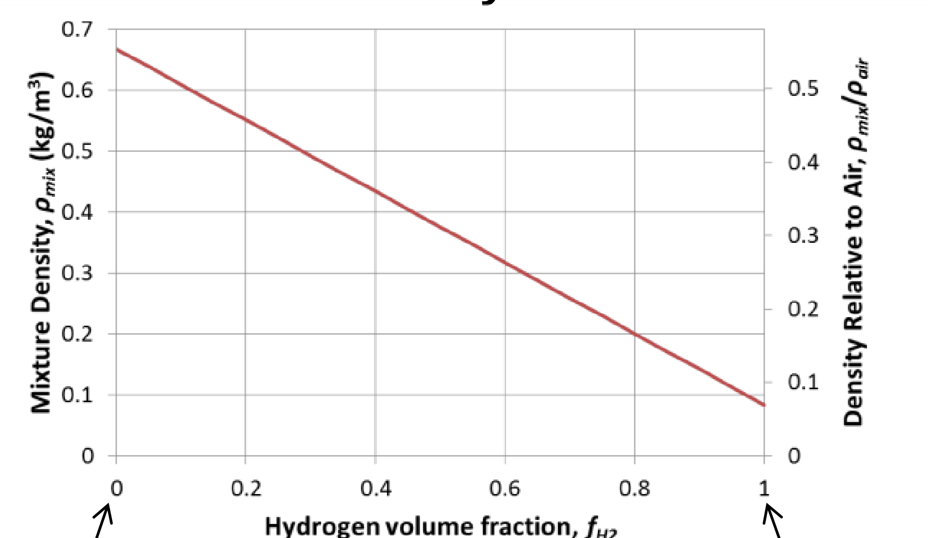
Gas Properties

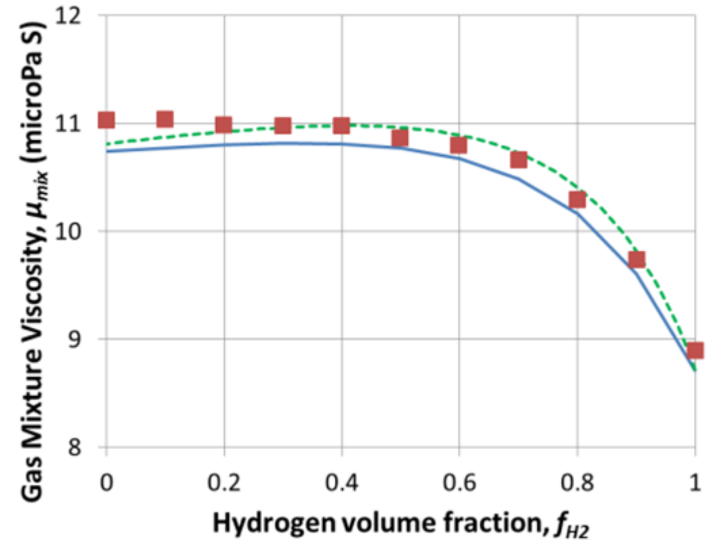
Density



Pure methane

Pure hydrogen

Viscosity



Viscosity graph:

▪▪▪ GasVLe model predictions using the Wilke-Brokaw formula with the Dean-Stiel density correction https://www.dnvgl.com/services/gasvle-8331

— Davidson, T.A. (1993) A simple and accurate method for calculating viscosity of gaseous mixtures, Report of Investigations 9456, US Bureau of Mines. https://stacks.cdc.gov/view/cdc/10045/cdc_10045_DS1.pdf

■ Kobayashi, Y., Kurokawa, A. and Hirata, M. (2007) Viscosity measurement of hydrogen-methane mixed gas for future energy systems, Journal of Thermal Science and Technology, 2(2), p236-244. https://doi.org/10.1299/jtst.2.236,

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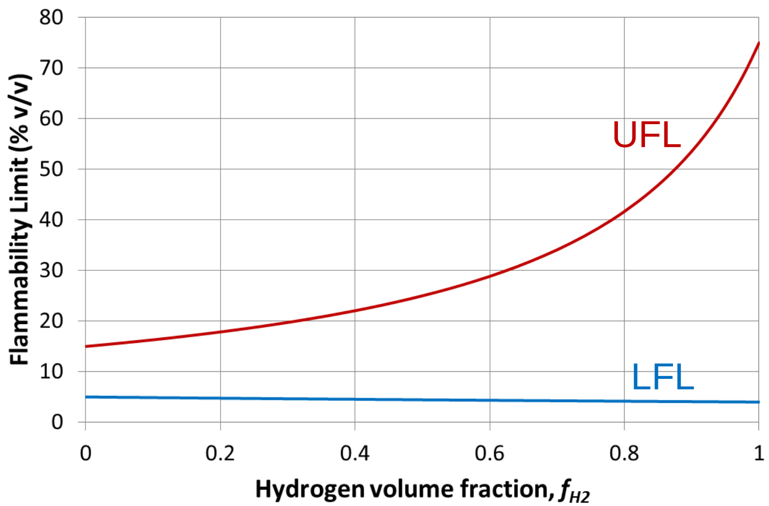
Gas Properties

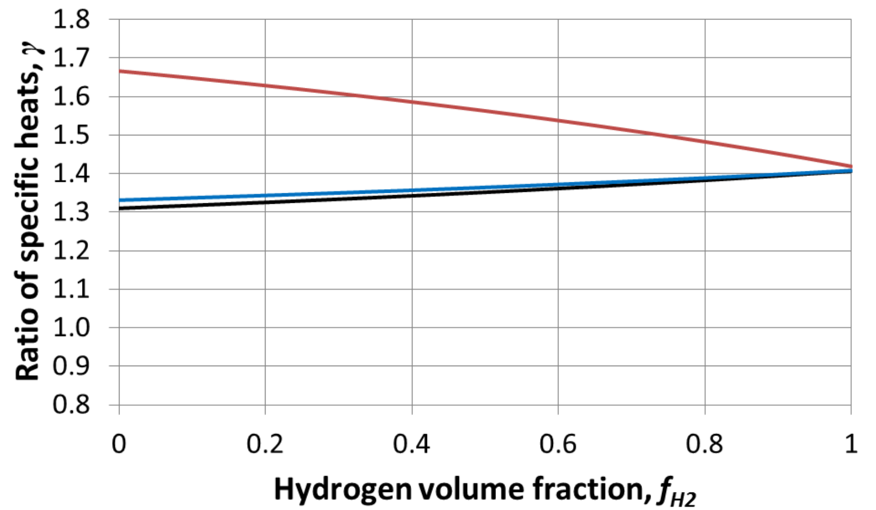
Flammability limits



Assumed LFL_{CH4} = 5.0 % v/v; LFL_{H2} = 4.0 % v/v

UFL_{CH4} = 15 % v/v; UFL_{H2} = 75 % v/v

Ratio of Specific Heat Capacities, γ



- 85 barg
- 7 barg
- standard atmospheric pressure

All at 15 °C

Sources:

https://encyclopedia.airliquide.com

https://webbook.nist.gov

Coward, H.F. and Jones, G.W. (1952) Limits of flammability of gases and vapors, US Bureau of Mines Bulletin 503
https://apps.dtic.mil/dtic/tr/fulltext/u2/701575.pdf

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Release Rate

Int. J. Hydrogen Energy, Vol. 17, No. 10, pp. 807–815, 1992.
Printed in Great Britain.

0360–3199/92 $5.00 + 0.00
Pergamon Press Ltd.
International Association for Hydrogen Energy.

A COMPARISON OF H_2, CH_4 AND C_3H_8 FUEL LEAKAGE IN RESIDENTIAL SETTINGS

M. R. SWAIN and M. N. SWAIN

Department of Mechanical Engineering, University of Miami, Coral Gables, FL 33124, U.S.A.

(*Received for publication* 19 *May* 1992)

Abstract—One of the boundary conditions for modeling gas cloud motion from a residential gas leak is the leakage rate at the source of the cloud. The literature presents estimates of the leakage rate of hydrogen relative to methane or propane based on diffusion, laminar flow and turbulent flow. These three methods yield significantly different values for the relative leakage rate of hydrogen. An experimental study was therefore conducted to measure the relative leakage rate of hydrogen compared with methane or propane for various leaks. The results from the experimental study were used as input to a computer model to predict combustible gas cloud size and motion in residential kitchens.

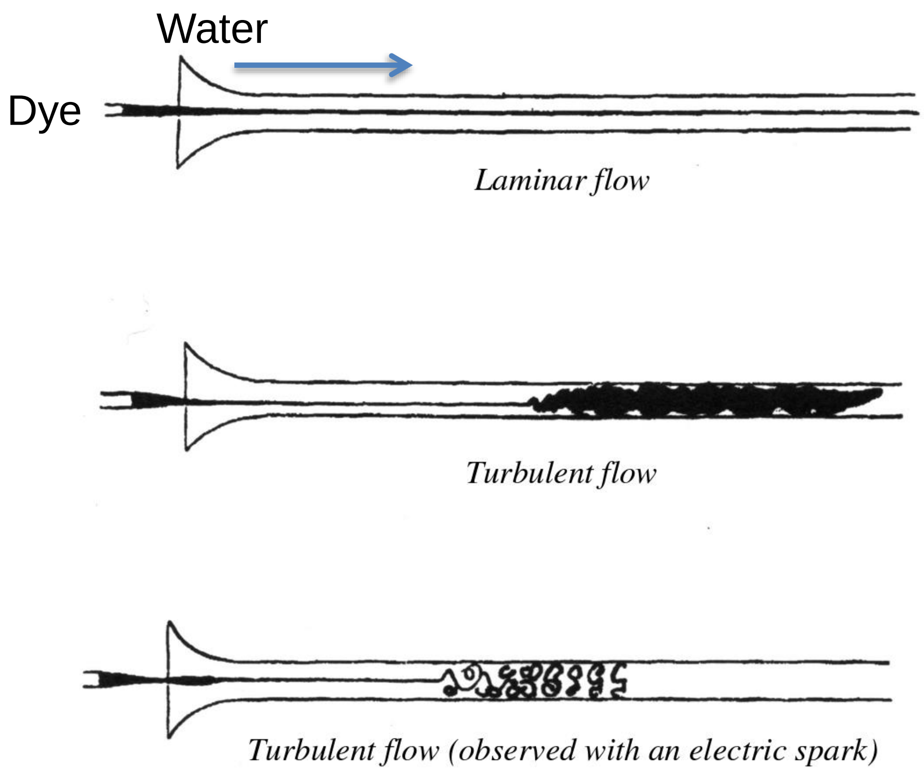
INTRODUCTION

Present concern about possible changes in climate due to the greenhouse effect has renewed interest in the use of hydrogen as a fuel. The analysis presented here was made to compare hydrogen leakage, from premise fuel lines (residential gas distribution systems), with that of methane or propane. Gas leakage in premise fuel lines increases energy consumption and sets the stage for accidental combustion.

where A is the cross-sectional area of the leak, D is the diffusion constant of the gas, and $\partial c/\partial x$ is the concentration gradient. If we compare the leakage rate of hydrogen with that of methane, using diffusion, the area for leakage is the same for both gases (same leak) and $\partial c/\partial x$ is defined by the length of the leak pathway which remains the same for each gas. Therefore, the ratio of molar flow rates between hydrogen and methane would be:

https://doi.org/10.1016/0360-3199(92)90025-R

Laminar and Turbulent Flow



Reynolds number

$$Re = \frac{\rho U D}{\mu}$$

ρ	Density
U	Velocity
D	Pipe diameter
μ	Dynamic viscosity

$Re < 2000$	Laminar
$2000 < Re < 4000$	Transitional
$Re > 4000$	Turbulent

https://en.wikipedia.org/wiki/Osborne_Reynolds

Release Rate

Laminar

Volume flow rate $$\dot{V}_{laminar} = \frac{\Delta P \pi D^4}{128 L \mu}$$

ΔP Pressure drop
D Leak diameter
L Leak path length
μ Dynamic viscosity

For the same pressure, leak diameter and leak length:

$$\frac{\dot{V}_{H2}}{\dot{V}_{CH4}} = \frac{\mu_{CH4}}{\mu_{H2}} = \frac{1.1 \times 10^{-5}}{8.7 \times 10^{-6}} = 1.23$$

Source:

Swain, M.R. and Swain, M.N. (1992) A comparison of H_2, CH_4 and C_3H_8 fuel leakage in residential settings, Int. J. Hydrogen Energy, 17(10), p807-815. https://doi.org/10.1016/0360-3199(92)90025-R

Release Rate

Turbulent

Volume flow rate $$\dot{V}_{turbulent} = 0.354\pi \frac{D^{2.5}\sqrt{\Delta P}}{\sqrt{fL\rho}}$$

ΔP Pressure drop
D Leak diameter
f Friction factor
L Leak path length
ρ Density

For the same pressure, leak diameter and leak length :

$$\frac{\dot{V}_{H2}}{\dot{V}_{CH4}} = \sqrt{\frac{\rho_{CH4}}{\rho_{H2}}} = \sqrt{\frac{M_{CH4}}{M_{H2}}} = \sqrt{\frac{16}{2}} = 2.8$$

Source:

Swain, M.R. and Swain, M.N. (1992) A comparison of H_2, CH_4 and C_3H_8 fuel leakage in residential settings, Int. J. Hydrogen Energy, 17(10), p807-815. https://doi.org/10.1016/0360-3199(92)90025-R

Release Rate

Subsonic, below approx. 0.9 barg

Mass flow rate* $\dot{m} = C_d A P \sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$

- C_d Discharge coeff.
- A Leak area
- P Pressure (abs.)
- M Molecular mass
- Z Compressibility
- R Ideal gas const.
- T Temperature
- γ Ratio of specific heat capacity

For the same pressure, leak diameter and leak length:

$$\frac{\dot{m}_{H2}}{\dot{m}_{CH4}} = C_{subsonic}\sqrt{\frac{M_{H2}}{M_{CH4}}} = \begin{cases} 1.000\sqrt{\frac{2}{16}} = 0.35 & \text{for } P = 21 \text{ mbarg} \\ 1.026\sqrt{\frac{2}{16}} = 0.36 & \text{for } P = 0.9 \text{ barg} \end{cases}$$

← Gas distribution pressure (for $P = 21$ mbarg)

← Choke pressure (for $P = 0.9$ barg)

*Source:

BS EN 60079-10-1. Explosive atmospheres, Part 10-1: Classification of areas – Explosive gas atmospheres, BSI Standards Publication, ISBN 978 0 580 96948 5, 2015.

Release Rate

Choked, above approx. 0.9 barg

Mass flow rate*

$$\dot{m} = C_d A P \sqrt{\gamma \frac{M}{ZRT} \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/(\gamma-1)}}$$

For the same pressure, leak diameter and leak length:

$$\frac{\dot{m}_{H2}}{\dot{m}_{CH4}} = C_{choked} \sqrt{\frac{M_{H2}}{M_{CH4}}} = 1.025 \sqrt{\frac{2}{16}} = 0.36$$

C_d	Discharge coeff.
A	Leak area
P	Pressure (abs.)
M	Molecular mass
Z	Compressibility
R	Ideal gas const.
T	Temperature
γ	Ratio of specific heat capacity

*Source:

BS EN 60079-10-1. Explosive atmospheres, Part 10-1: Classification of areas – Explosive gas atmospheres, BSI Standards Publication, ISBN 978 0 580 96948 5, 2015.

Release Rate

Converting between volume, mass and energy

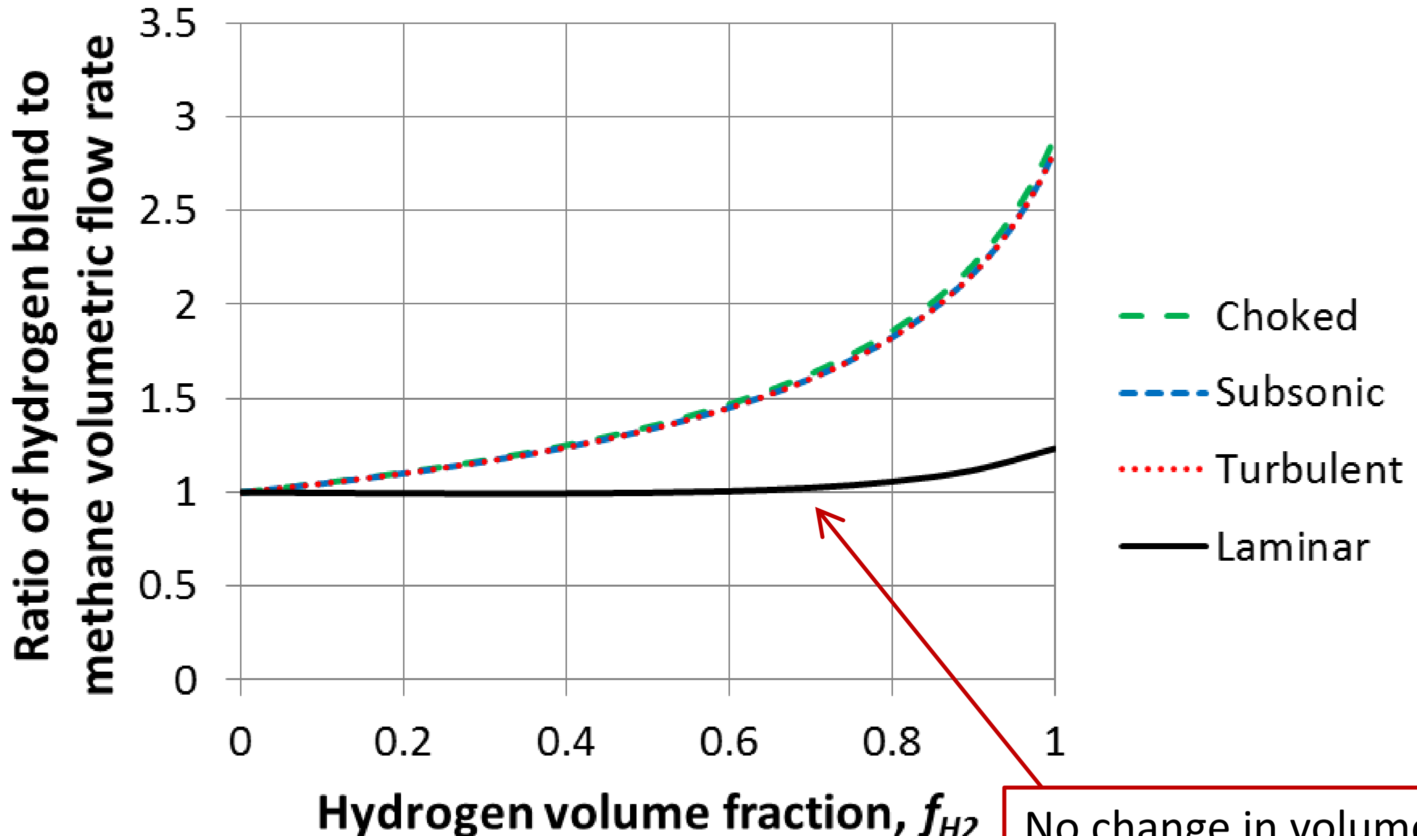
Mass $$\frac{\dot{m}_{H2}}{\dot{m}_{CH4}} = \frac{M_{H2}}{M_{CH4}}\frac{\dot{V}_{H2}}{\dot{V}_{CH4}} = \frac{2}{16}1.23 = 0.15$$

Energy $$\frac{\dot{Q}_{H2}}{\dot{Q}_{CH4}} = \frac{Q_{H2}}{Q_{CH4}}\frac{\dot{V}_{H2}}{\dot{V}_{CH4}} = \frac{285.8}{890.8}1.23 = 0.40$$

Q Heat of combustion

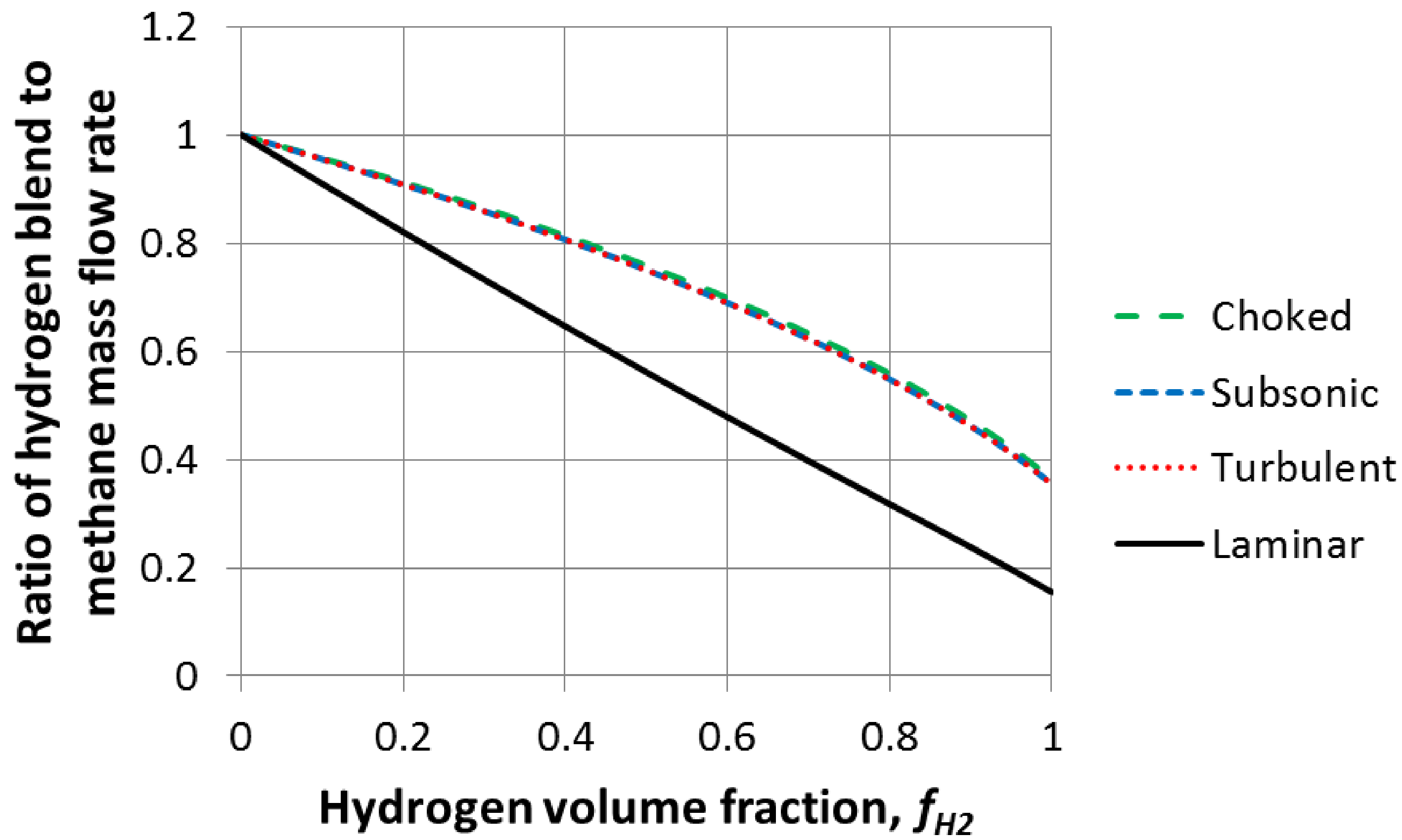
- "Gross" heats of combustion used here, i.e. water produced in the combustion reaction is condensed into liquid, and the heat of combustion value accounts for the resulting release of latent heat
- To calculate the heat released in a fire, it is more appropriate to use the "net" heat of combustion, which is 5 – 15% lower, since water remains as vapour in that case. Also need to consider combustion efficiency and radiative heat fraction. Such analysis is left for future work

Volumetric Release Rate

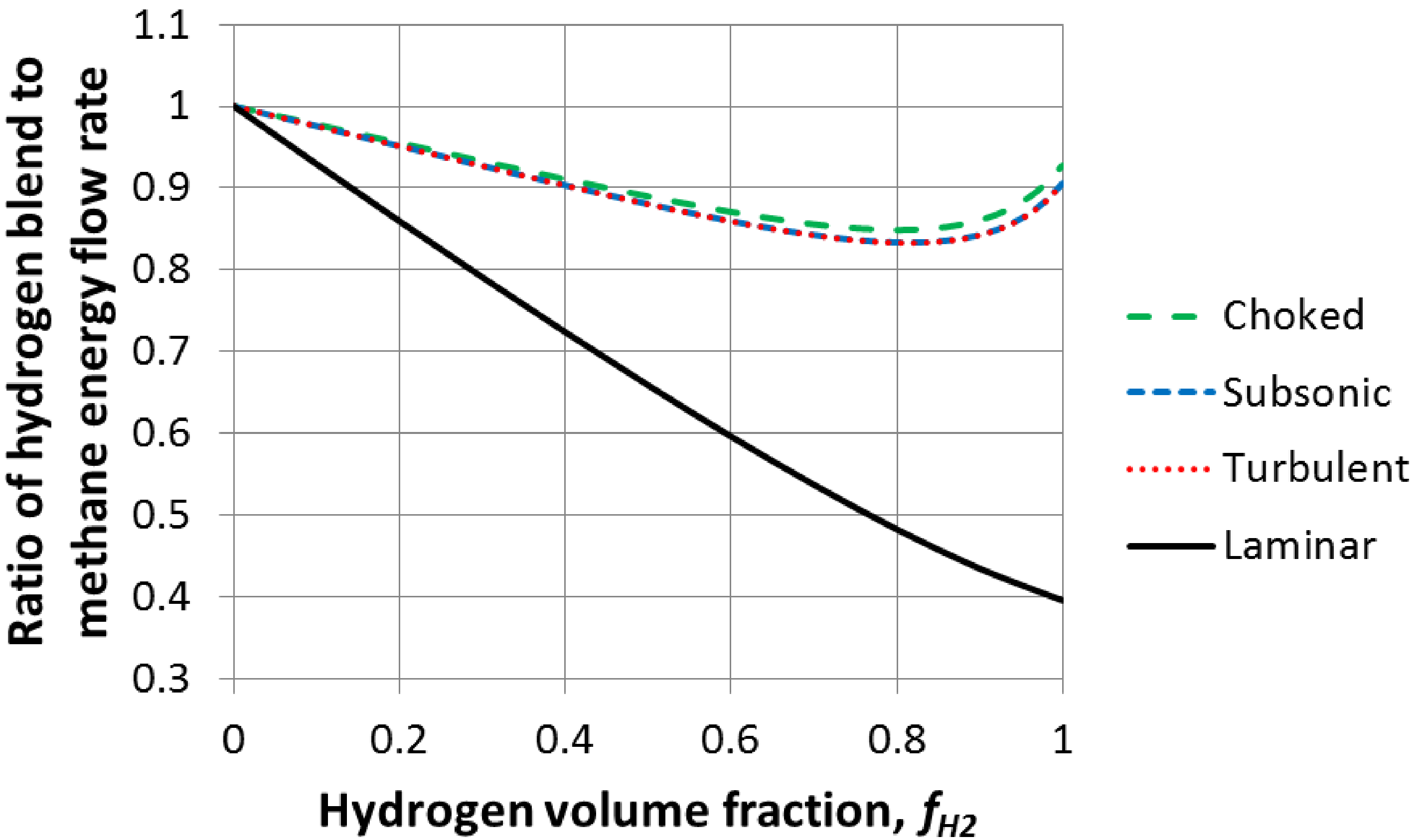


No change in volumetric flow rate for laminar leaks up to around 70% hydrogen

Mass Release Rate



Energy Release Rate



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Dispersion

ISBN 0-08-024772-5

VERTICAL TURBULENT BUOYANT JETS
A Review of Experimental Data

By
CHING JEN CHEN
Iowa Institute of Hydraulic Research
The University of Iowa, Iowa City, Iowa 52242, U.S.A.
And
WOLFGANG RODI
Sonderforschungsbereich 80
Universität Karlsruhe, Karlsruhe, Germany

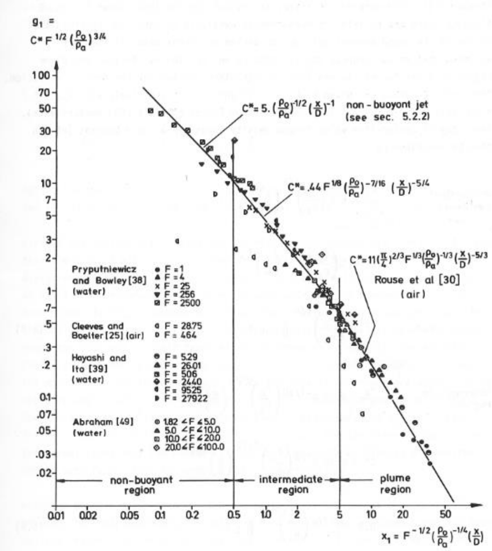

Fig. 5. Decay of center-line density in axisymmetric buoyant jets

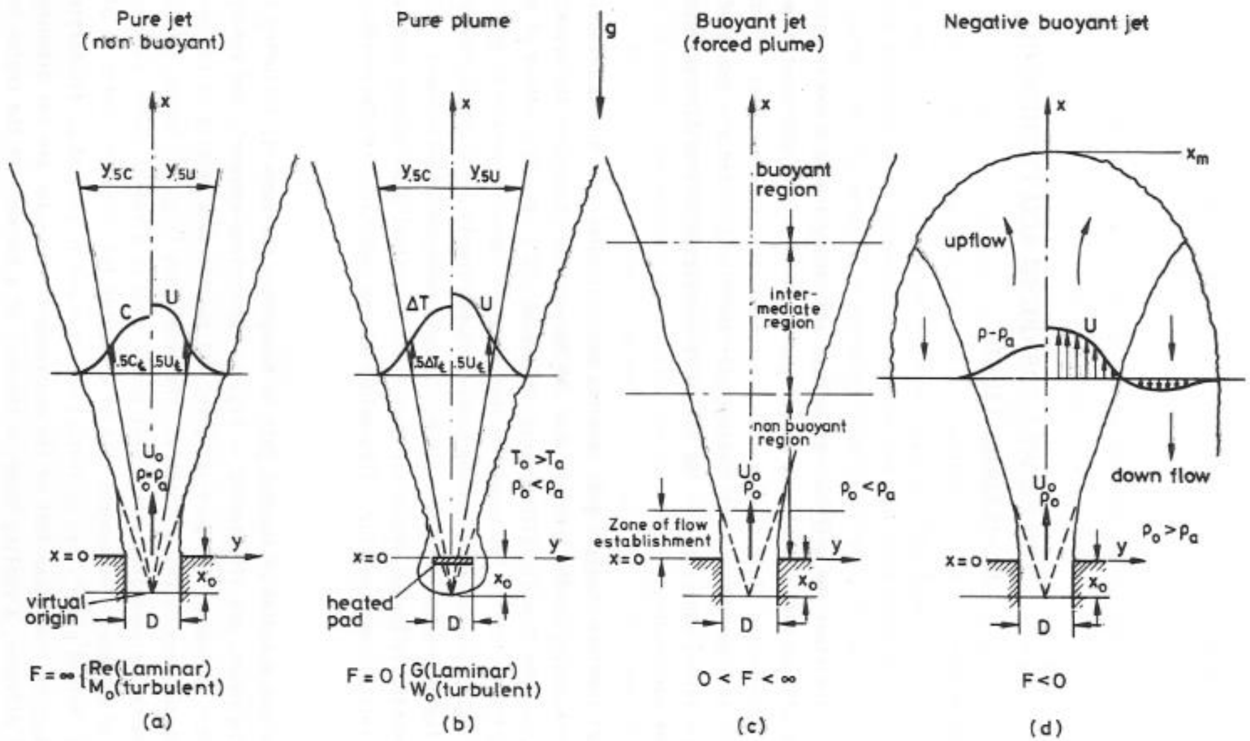

Fig. 1. Buoyant jets in uniform surroundings

Dispersion

Jets

Gas concentration (mass fraction)

$$y = k\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D}{x}$$

k	Model constant
ρ_0	Gas density
ρ_a	Air density
D	Leak diameter
x	Axial distance

$$\frac{\text{Distance to LFL for hydrogen}}{\text{Distance to LFL for methane}} =$$

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{\rho_{H2}}{\rho_{CH4}}\right)^{\frac{1}{2}}\frac{y_{CH4}}{y_{H2}} = \left(\frac{M_{H2}}{M_{CH4}}\right)^{\frac{1}{2}}\frac{y_{CH4}}{y_{H2}} = \left(\frac{2}{16}\right)^{\frac{1}{2}}\frac{2.8}{0.29} = 3.5$$

↑ LFLs expressed as mass fractions (% w/w)

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Dispersion

Intermediate Jet-Plume

Gas concentration (volume fraction) $$C^* = 4.4Fr^{\frac{1}{8}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{7}{16}}\left(\frac{x}{D}\right)^{-\frac{5}{4}}$$

$$Fr = \frac{\rho_0 U_0^2}{gD(\rho_a - \rho_0)}$$

Fr	Froude number
ρ_0	Gas density
ρ_a	Air density
x	Axial distance
D	Leak diameter
U_0	Exit velocity
g	Gravitational accel.

Buoyant plumes

Gas concentration (volume fraction) $$C^* = 9.35Fr^{\frac{1}{3}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{3}}\left(\frac{x}{D}\right)^{-\frac{5}{3}}$$

Dispersion

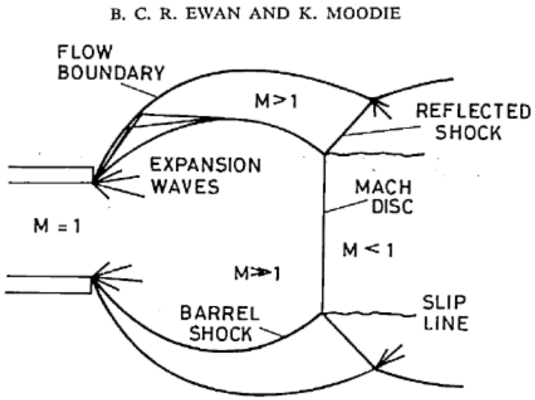
For **choked** jet releases

Gas concentration (mass fraction)

$$y = k\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D_{eff}}{x+a}$$

Effective source diameter, D_{eff}

Effective source offset distance, a



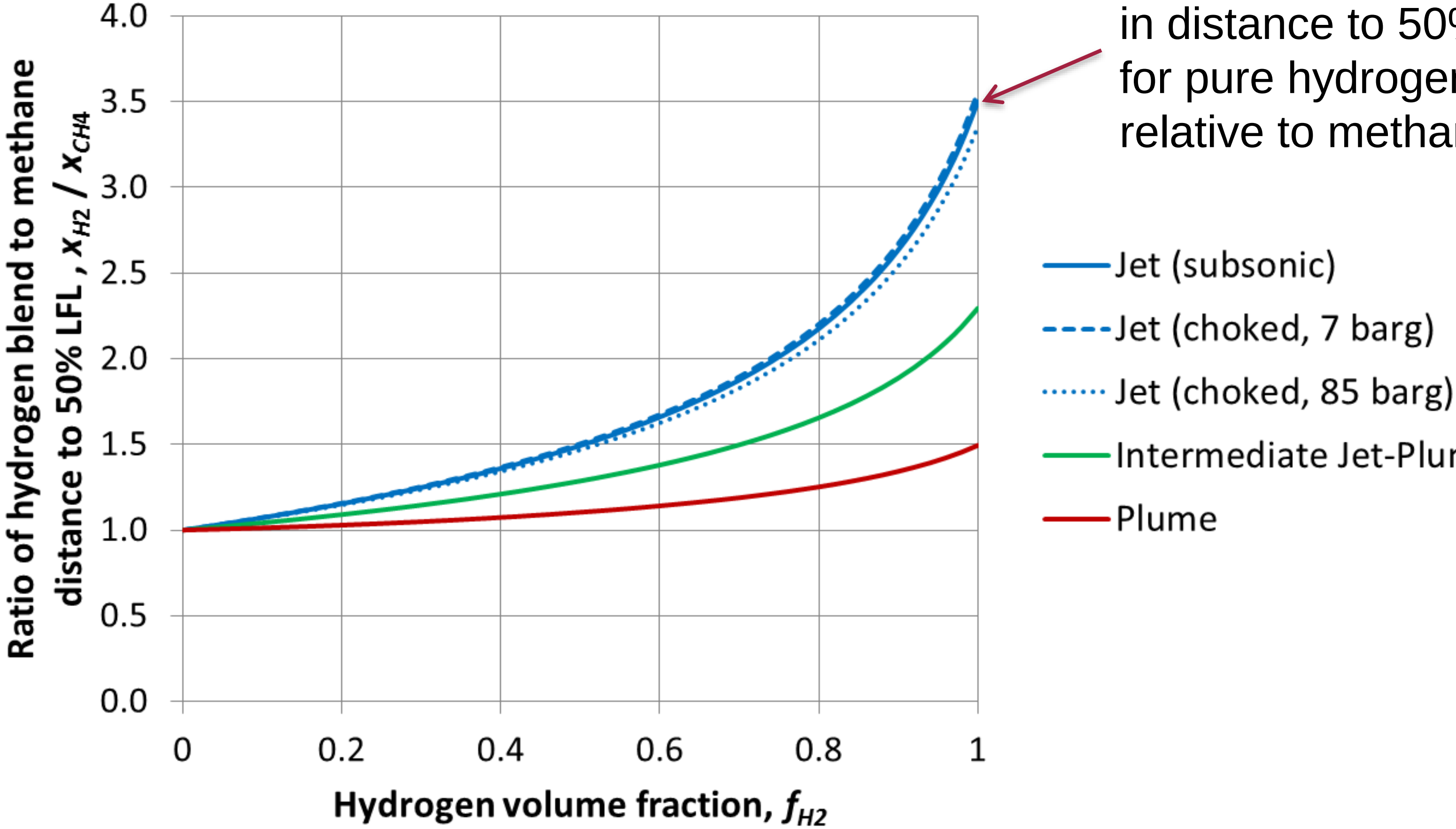
1.) Birch, A.D., Hughes, D.J., Swaffield, F., 1987. Velocity decay of high pressure jets, Combust. Sci. and Tech. 52, 161-171, https://doi.org/10.1080/00102208708952575

$$\frac{D_{eff}}{D} = C_D\sqrt{\left[\frac{P}{P_{atm}}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\frac{\gamma}{(\gamma C_D^2+1)}\right]}$$

2.) Ewan, B.C.R and Moodie, K., 1986. Structure and velocity measurements in underexpanded jets, Combust. Sci. and Tech. 45, 275-288, https://doi.org/10.1080/00102208608923857

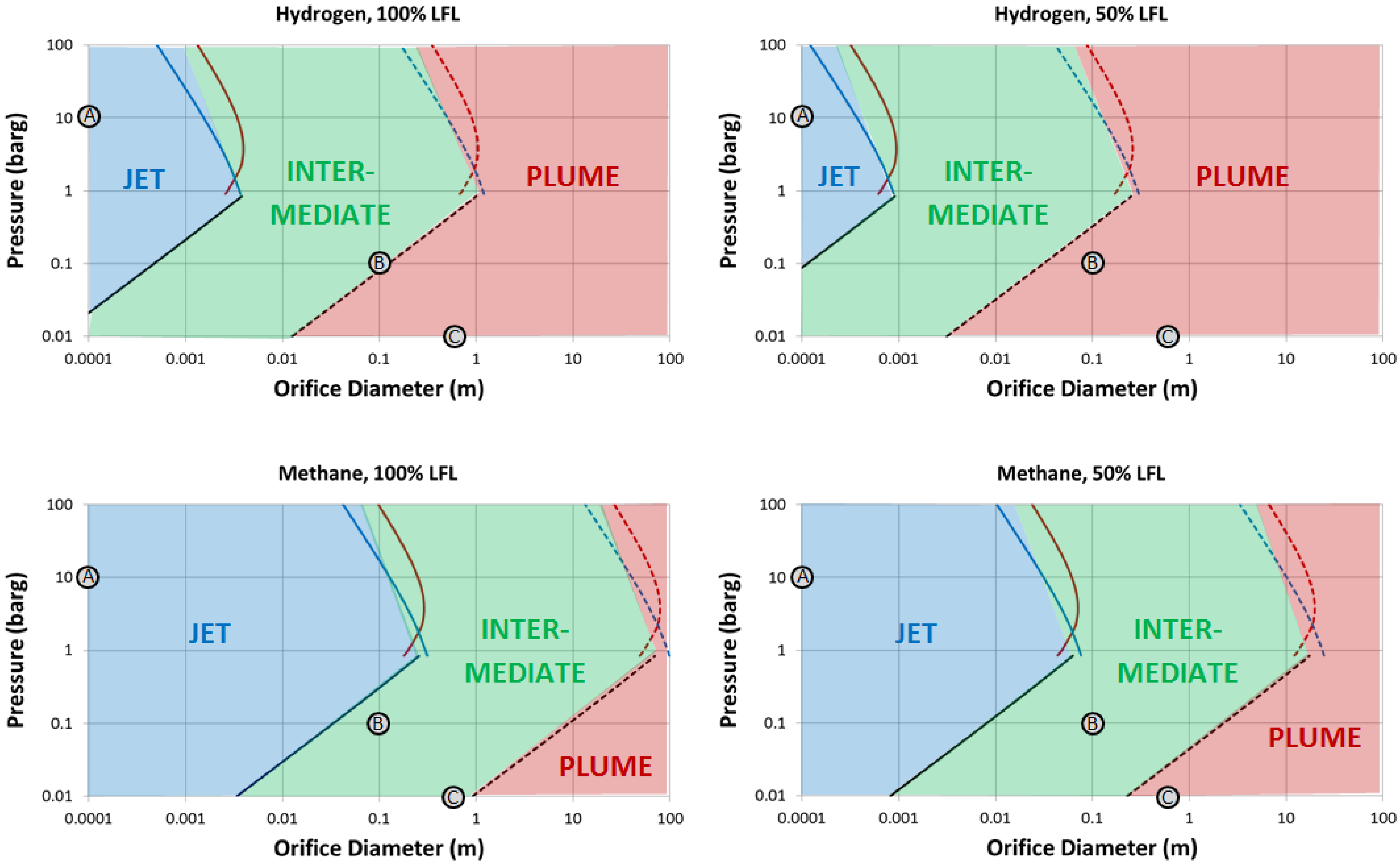
$$\frac{D_{eff}}{D} = \left(\frac{P_e}{P_a}\right)^{\frac{1}{2}} \quad ; \quad P_e = P\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

Dispersion



Factor of 3.5 increase in distance to 50% LFL for pure hydrogen relative to methane

When is it a jet or a plume?



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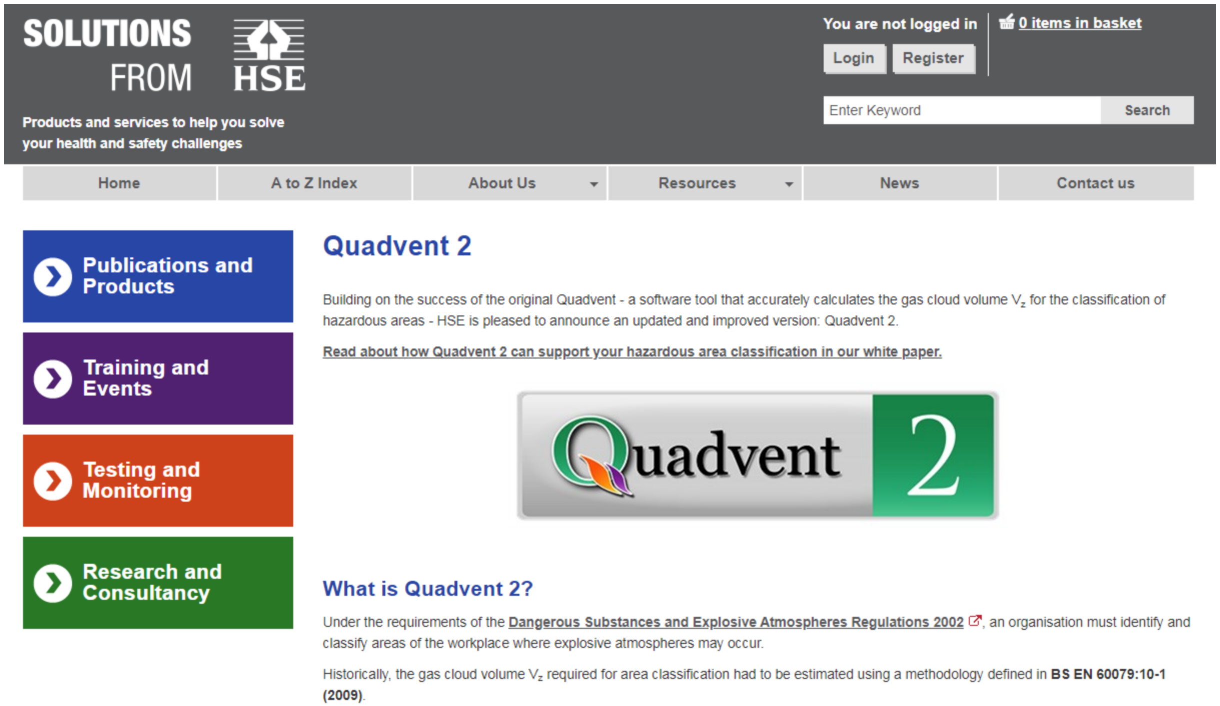
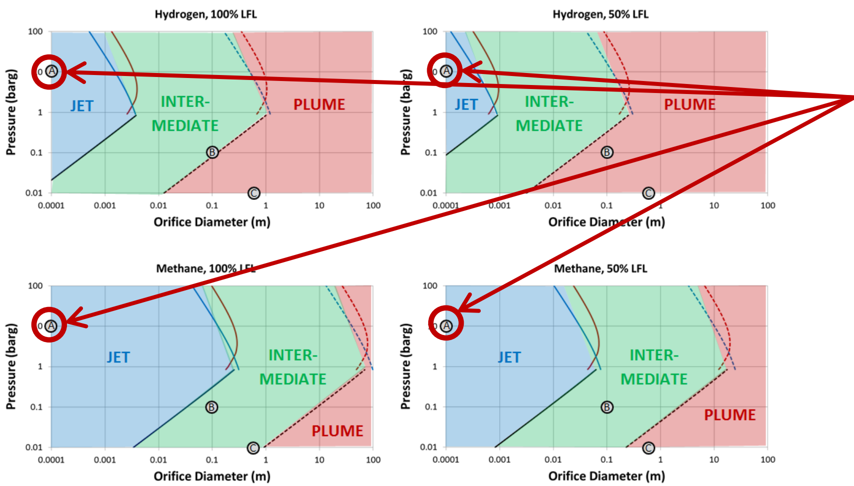
Quadvent model



https://www.hsl.gov.uk/publications-and-products/quadvent-2

Webber, D.M., Ivings, M.J. and Santon, R.C., 2011. Ventilation theory and dispersion modelling applied to hazardous area classification, J. Loss Prev. Process Ind. 24(5), 612-621. https://doi.org/10.1016/j.jlp.2011.04.002

Comparison to Quadvent model



Point A

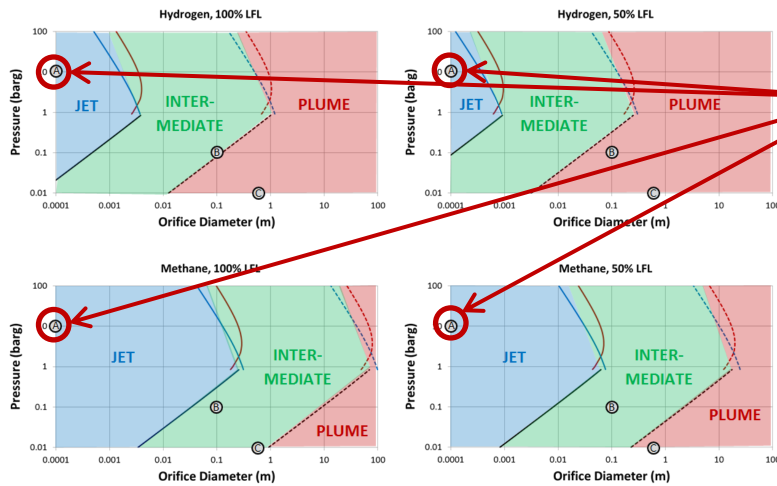
Pressure = 10 barg

Orifice diameter = 0.1 mm

Quadvent

	100% LFL	50% LFL
Hydrogen distance (x_{H2})	0.11 m	0.22 m
Methane distance (x_{CH4})	0.030 m	0.062 m
Ratio of hydrogen to methane distances (x_{H2}/x_{CH4})	3.6	3.6

Comparison to Quadvent model



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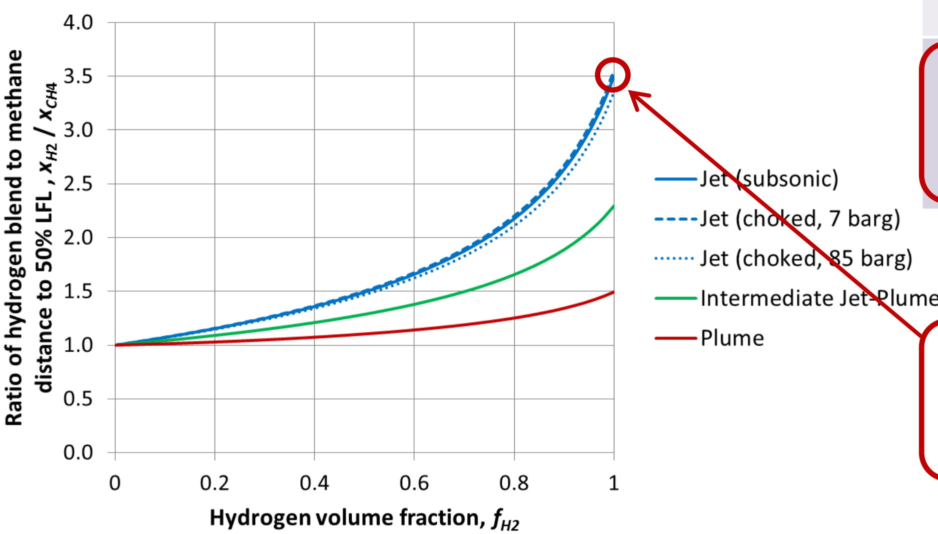

Chen and Rodi (1980) for jets gives (x_{H2}/x_{CH4}) = 3.5 to 3.6

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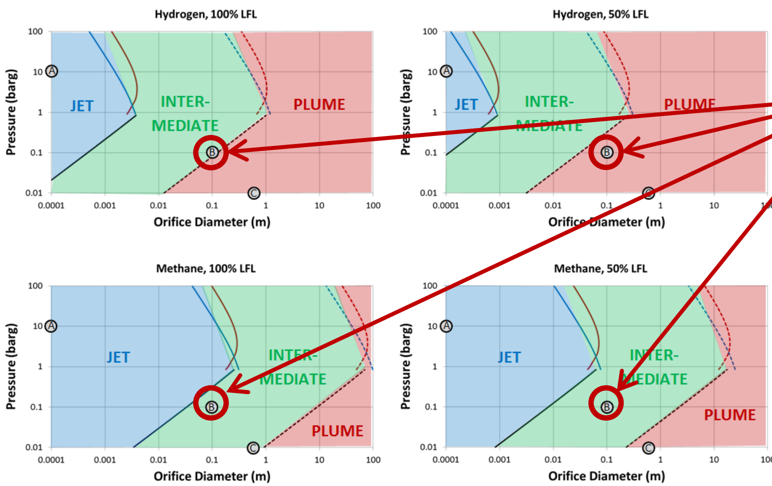
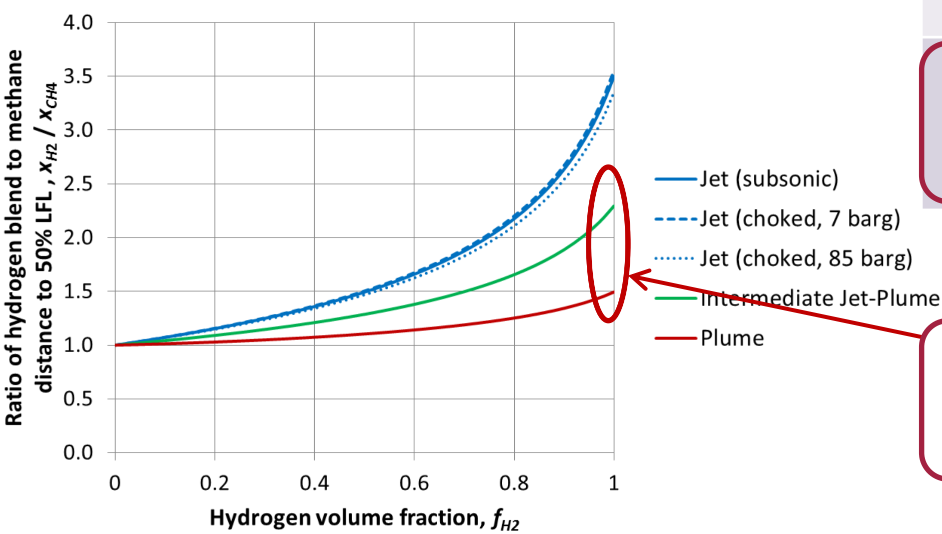
Comparison to Quadvent model



Point B
Pressure = 0.1 barg
Orifice diameter = 100 mm

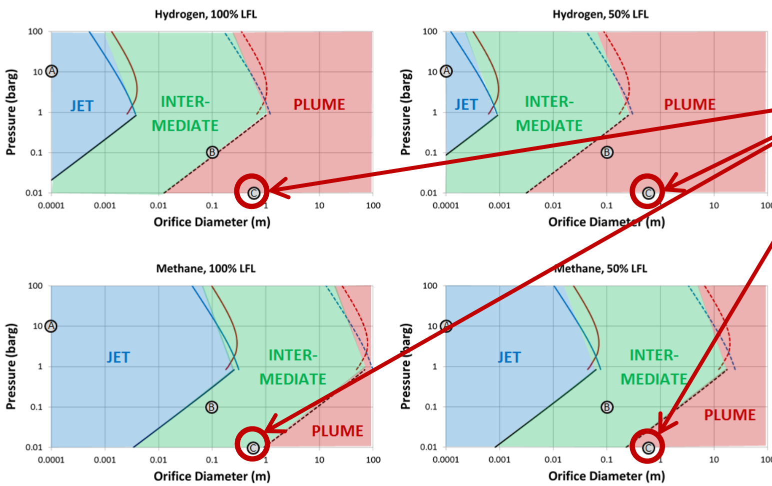
Quadvent

	100% LFL	50% LFL
Hydrogen distance (x_{H2})	34 m	57 m
Methane distance (x_{CH4})	12 m	23 m
Ratio of hydrogen to methane distances (x_{H2}/x_{CH4})	2.9	2.4



Chen and Rodi (1980) for intermediate and plumes gives (x_{H2}/x_{CH4}) = 2.3 to 1.5

Comparison to Quadvent model



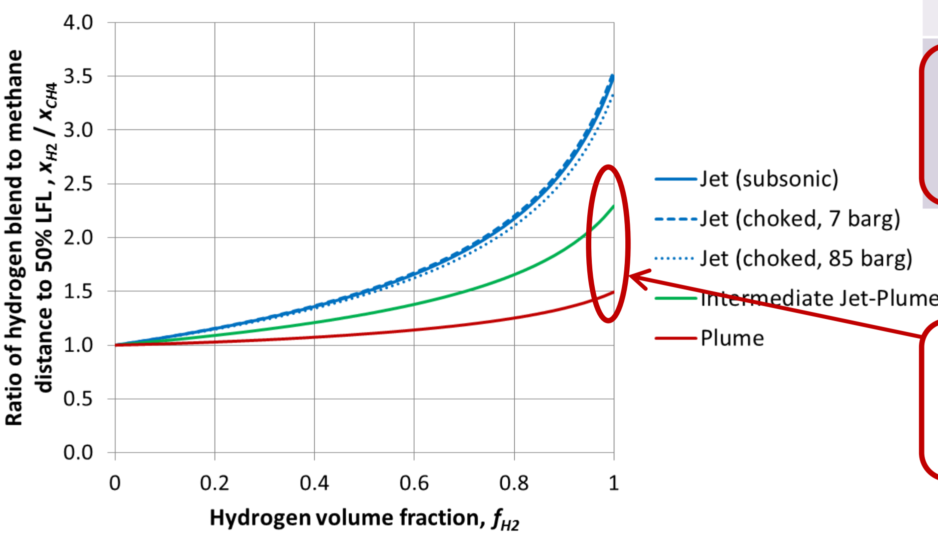
Point C

Pressure = 0.01 barg

Orifice diameter = 500 mm

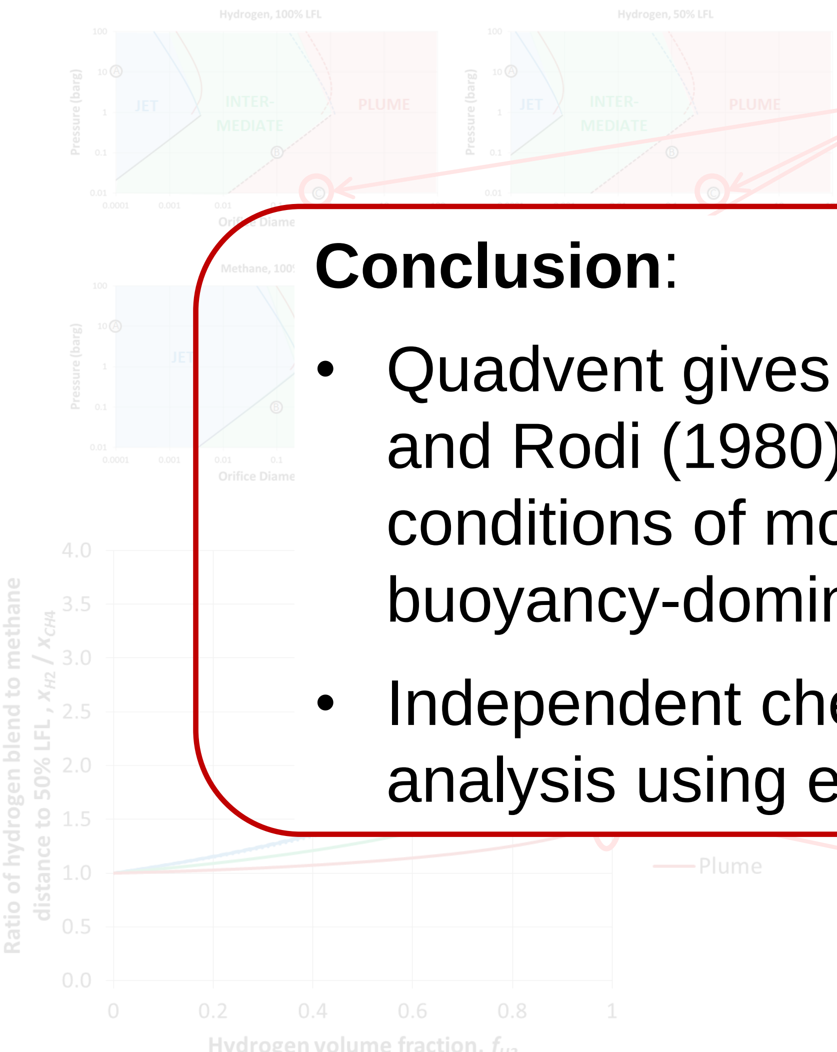
Quadvent

	100% LFL	50% LFL
Hydrogen distance (x_{H2})	93 m	146 m
Methane distance (x_{CH4})	49 m	84 m
Ratio of hydrogen to methane distances (x_{H2}/x_{CH4})	1.9	1.7



Chen and Rodi (1980) for plumes and intermediate gives (x_{H2}/x_{CH4}) = 1.5 to 2.3

Comparison to Quadvent model



Point C
Pressure = 0.01 barg
Orifice diameter = 500 mm

Quadvent

50% LFL
146 m
84 m
1.7

Conclusion:

- Quadvent gives similar results to the Chen and Rodi (1980) correlations for the limiting conditions of momentum-dominated jets and buoyancy-dominated plumes
- Independent check supports the earlier analysis using empirical correlations

Chen and Rodi (1980) for plumes and intermediate gives (x_{H2}/x_{CH4}) = 1.5 to 2.3

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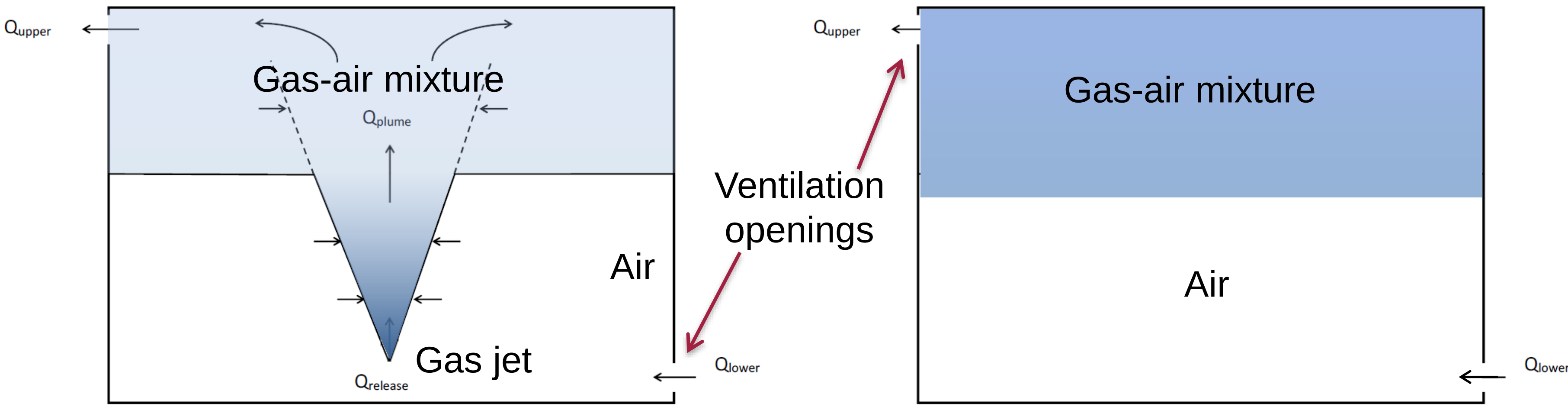
Gas accumulation models

NaturalHy project



Lowesmith, B.J., Hankinson, G., Spataru, C. and Stobhart, M. (2009) Gas build-up in a domestic property following releases of methane/hydrogen mixtures, Int. J. Hydrogen Energy, 34(14), p5932-5939. https://repository.lboro.ac.uk/articles/Gas_build-up_in_a_domestic_property_following_releases_of_methane_hydrogen_mixtures/9242582

HSE simplified model:

- Fully-mixed upper layer
- No jet model
- No wind: buoyancy drives flow

IGE/UP/1

Institution of gas Engineers & Managers

Utilization Procedures
IGE/UP/1 Edition 2
Reprint with Amendments
August 2005
Communication 1716

Founded 1863
Royal Charter 1929
Patron
Her Majesty the Queen

Strength testing, tightness testing and direct purging of industrial and commercial gas installations

https://www.igem.org.uk/technical-services/technical-gas-standards/utilisation/ige-up-1-edition-2-a-2005-strength-testing-tightness-testing-direct-purging-of-industrial-and-commercial-gas-installations/

IGE/UP/1

IGE/UP/1 Edition 2 – Reprint with Amendments. August 2005

5.5 MAXIMUM PERMITTED LEAK RATE (MPLR)

MPLR is the maximum permitted leak rate of the operating gas when the pipework is at OP. For a new installation, MPLR is a fixed value dependent only on the operating gas and is based on an energy release rate of 0.054 MJh^{-1}. Hence, MPLR for other gases not included in the tables can be calculated (see A4.1). The tightness test may use a different test gas and/or be at a pressure other than OP. Both of these will affect the measured leakage. The tables included in this section account for this for the common range of fuel gases and specified test pressures. Separate calculations would be needed for other gases or different pressures (see Appendix 4 or 5).

5.5.1 New installations and extensions

MPLR as shown in Table 7 shall apply irrespective of the location of the pipework.

Note: The prescribed test procedures will be capable of detecting the leakage rates shown.

GAS TYPE	MPLR ($m^3 h^{-1}$) at OP
NATURAL	0.0014
BUTANE	0.00044
PROPANE	0.00057
LPG/AIR (SNG)	0.0013
LPG/AIR (SMG)	0.0021
COAL GAS	0.0029

TABLE 7 - MPLR (NEW INSTALLATIONS AND EXTENSIONS)

IGE/UP/1

Based on previous analysis, can we answer these questions?

1. Is a leak of gas at the MPLR laminar or turbulent?
2. For an installation with a natural gas leak equal to the MPLR, how would the leak rate change if the gas was switched to hydrogen? What would be the implications in terms of flammable cloud size?
3. The MPLR for different gases is based on equivalent energy content (in MJ/hr). What would be the MPLR for hydrogen using this approach? What would be the implications in terms of the flammable cloud size?

Is the MPLR Laminar or Turbulent?

Methane MPLR volumetric flow rate		0.0014 m^3/hr
Laminar flow calculation		
	Methane MPLR hole diameter*	0.095 mm
	Methane MPLR Reynolds number	330
Subsonic flow calculation		
	Methane MPLR hole diameter	0.080 mm
	Methane MPLR Reynolds number	395

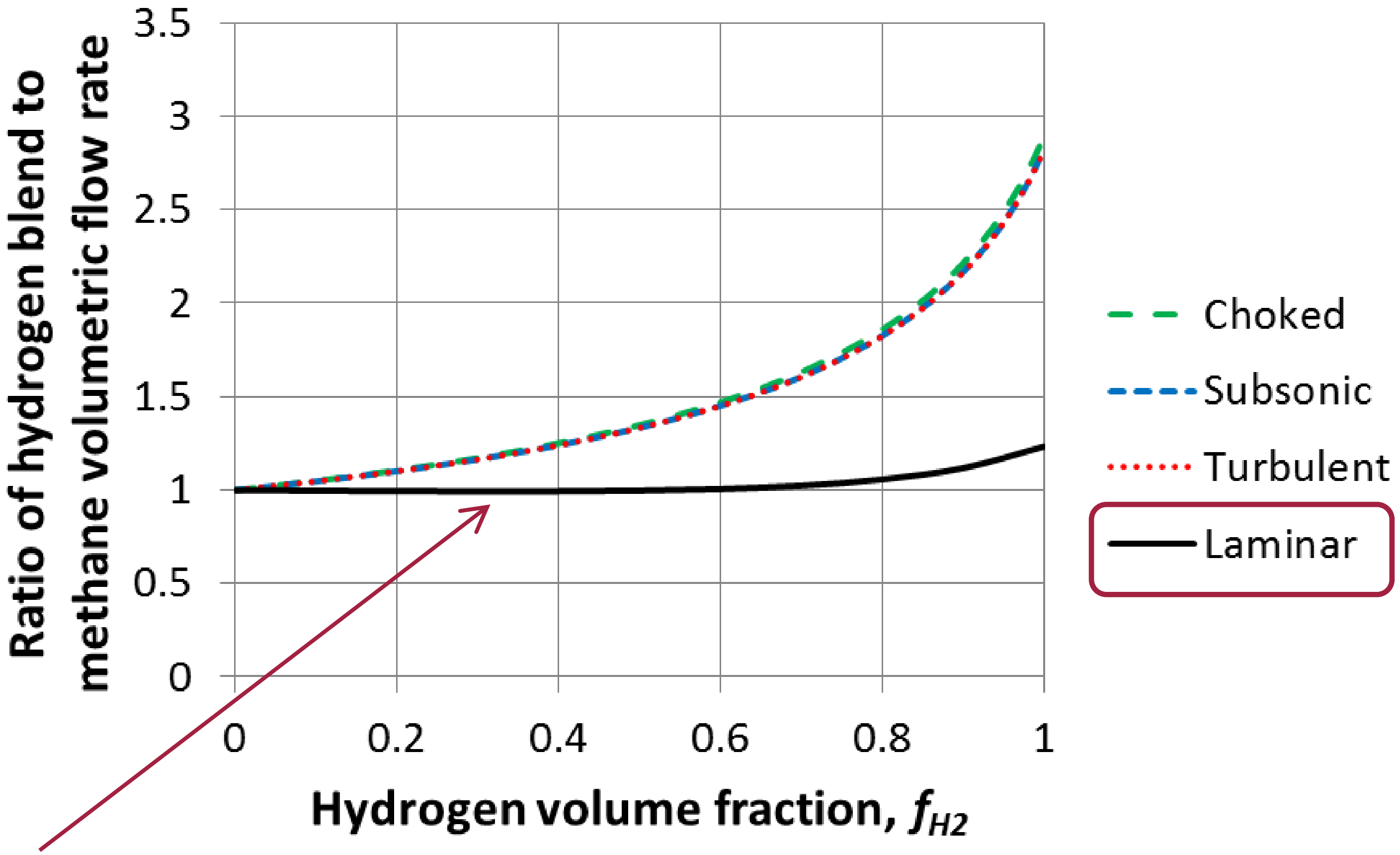
Assumed pressure = 21 mbarg

$$\dot{V}_{laminar} = \frac{\Delta P \pi D^4}{128 L \mu}$$

$$\dot{m}_{subsonic} = C_d A P \sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$$

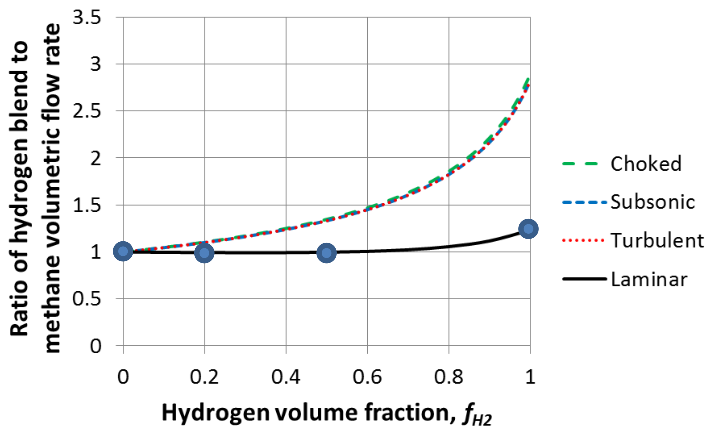
Conclusion: Flow through leak is laminar

Volumetric Release Rate



Volume flow rate unchanged with up to 70% hydrogen

IGE/UP/1: Scenario 1 calculations



Scenario 1

Flow rates calculated using 0.095 mm hole size that gives MPLR flow rate for methane at pressure of 21 mbarg

	Methane	20% Hydrogen	50% Hydrogen	100% Hydrogen
Lower flammability limit (% v/v)	5.0	4.8	4.4	4.0
Volumetric flow rate for the hole diameter calculated for the methane MPLR (0.095 mm) assuming laminar flow (m^3/hr) and pressure of 21 mbarg	0.0014	0.0014	0.0014	0.0017

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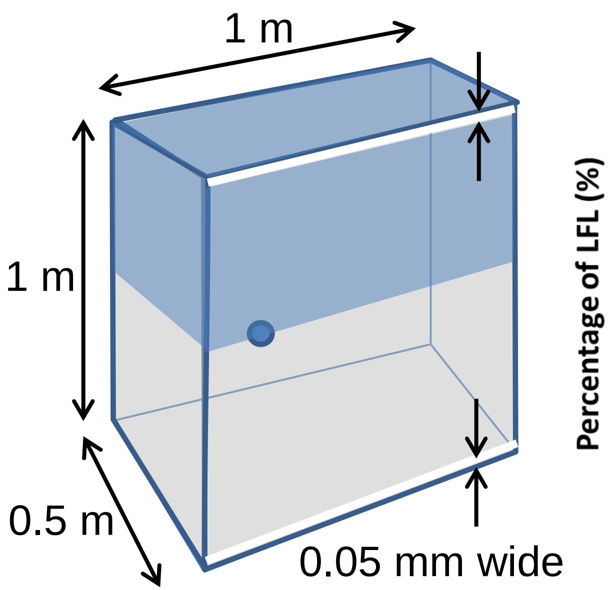
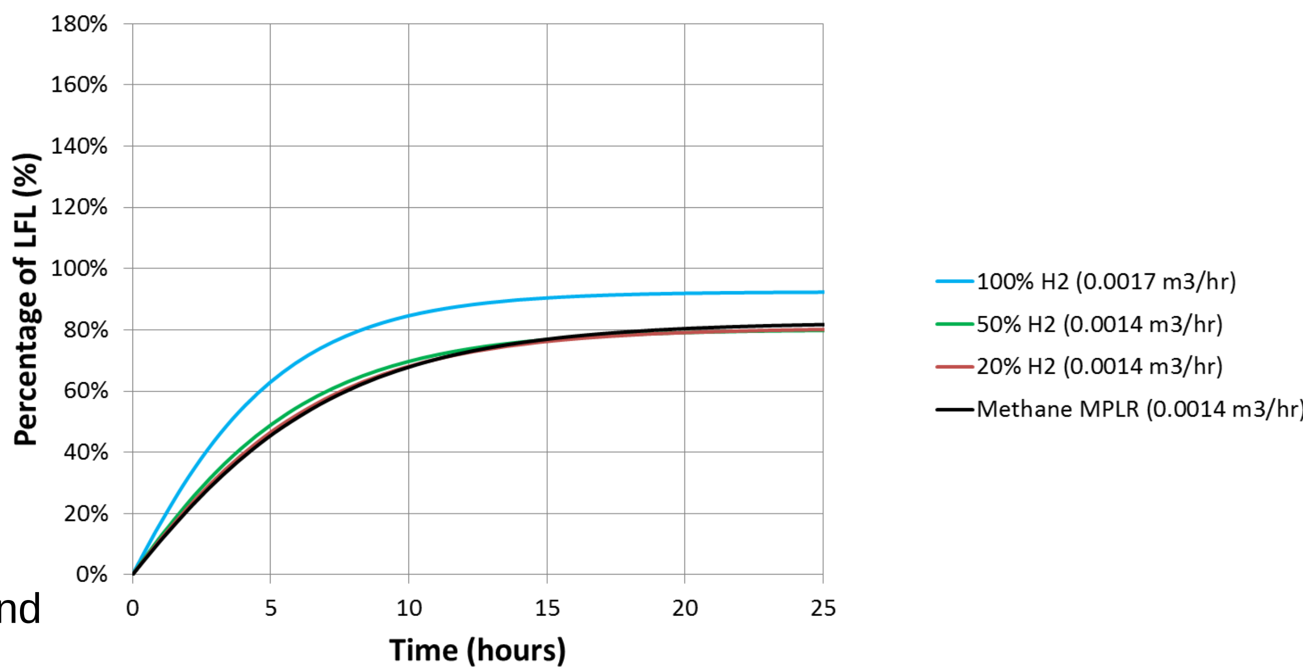
Gas Accumulation Results (Scenario 1)

Flow rates calculated using 0.095 mm hole size that gives MPLR flow rate for methane (pressure = 21 mbarg)





Gas leak in metering cupboard

Conclusion: for gas installations tested to IGE/UP/1 with natural gas, the calculation shows there is no increased risk of forming a flammable cloud from adding 20% or 50% hydrogen into natural gas

IGE/UP/1: Scenario 2 calculations

Scenario 2

Calculate flow rates from 0.054 MJ/hr

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5.5 **MAXIMUM PERMITTED LEAK RATE (MPLR)**

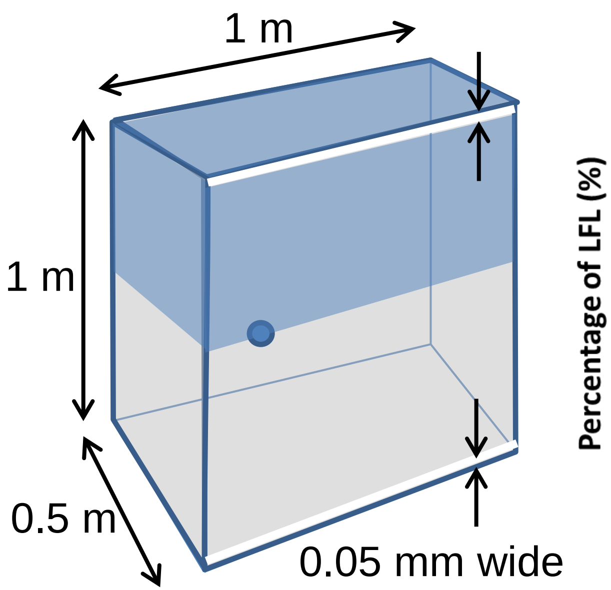
MPLR is the maximum permitted leak rate of the operating gas when the pipework is at OP. For a new installation, MPLR is a fixed value dependent only on the operating gas and is based on an energy release rate of 0.054 MJh^{-1}. Hence, MPLR for other gases not included in the tables can be calculated (see A4.1). The tightness test may use a different test gas and/or be at a pressure

	Methane	20% Hydrogen	50% Hydrogen	100% Hydrogen
Lower flammability limit (% v/v)	5.0	4.8	4.4	4.0
Gross heat of combustion (MJ/m^3)	37.7	32.6	24.9	12.1
Volumetric flow rate that gives an energy flow rate of 0.054 MJ/hr (m^3/hr) – Scenario 2	0.0014	0.0017	0.0022	0.0045

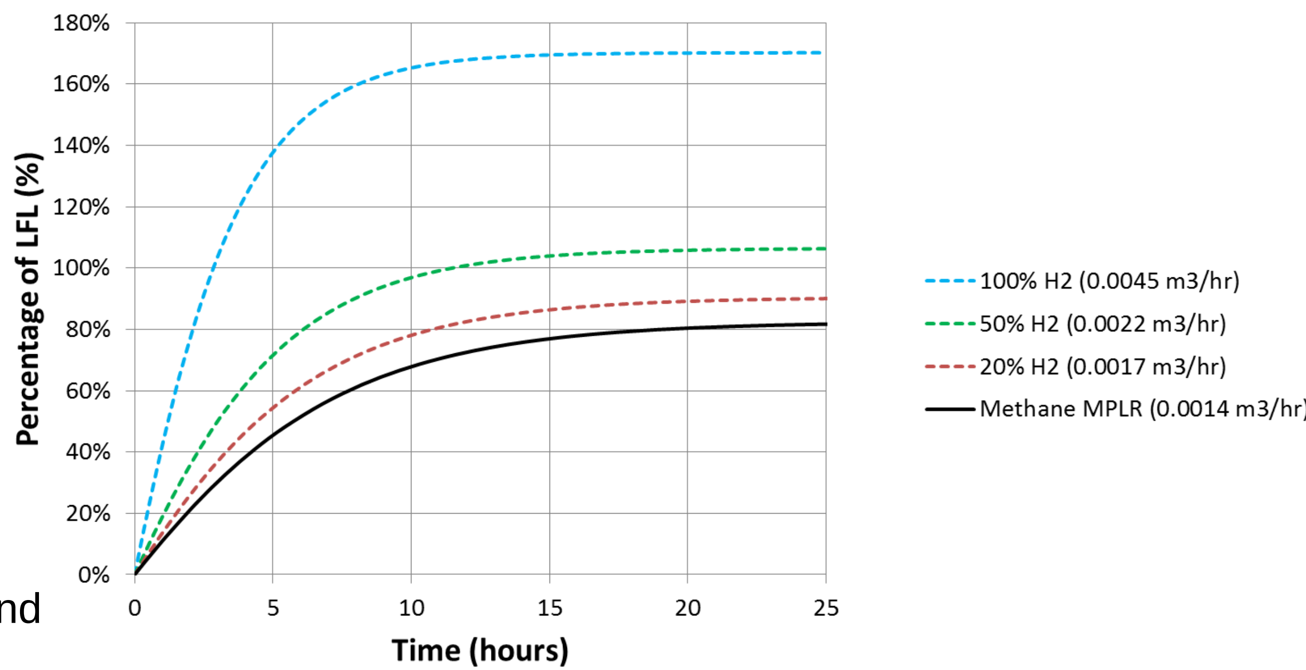
Increased flow rates with increased hydrogen content due to low heat of combustion per unit volume for hydrogen

Gas Accumulation Results (Scenario 2)

Flow rates calculated using MPLR energy release rate of 0.054 MJ/hr



0.05 mm wide opening top and bottom



Conclusion: increased risk of forming a flammable cloud if hydrogen blend MPLR is calculated from 0.054 MJ/hr

Gas leak in metering cupboard

IGE/UP/1: Scenario 3 calculations

GAS TYPE	MPLR ($m^3 h^{-1}$) at OP
NATURAL	0.0014
BUTANE	0.00044
PROPANE	0.00057
LPG/AIR (SNG)	0.0013
LPG/AIR (SMG)	0.0021
COAL GAS	0.0029

TABLE 7 - MPLR (NEW INSTALLATIONS AND EXTENSIONS)

Scenario 3

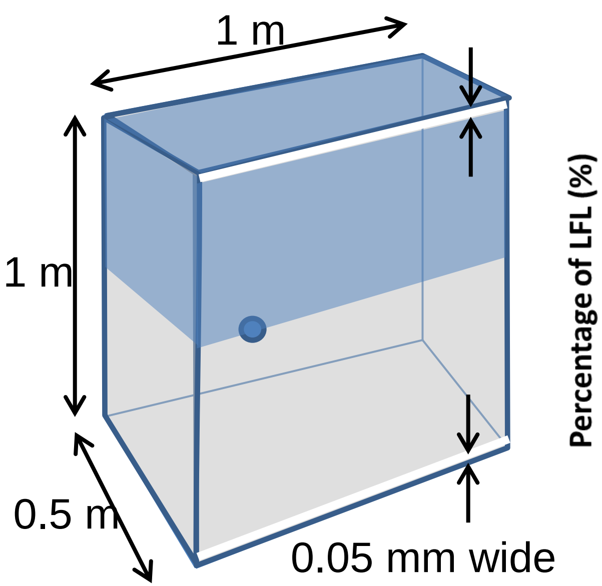
What would happen if we fixed the MPLR for hydrogen blends to be the same as that for natural gas (i.e. 0.0014 m^3/hr)?

	Methane	20% Hydrogen	50% Hydrogen	100% Hydrogen
Lower flammability limit (% v/v)	5.0	4.8	4.4	4.0
Volumetric flow rate that gives an energy flow rate of 0.054 MJ/hr (m^3/hr) – Scenario 2	0.0014	0.0014	0.0014	0.0014

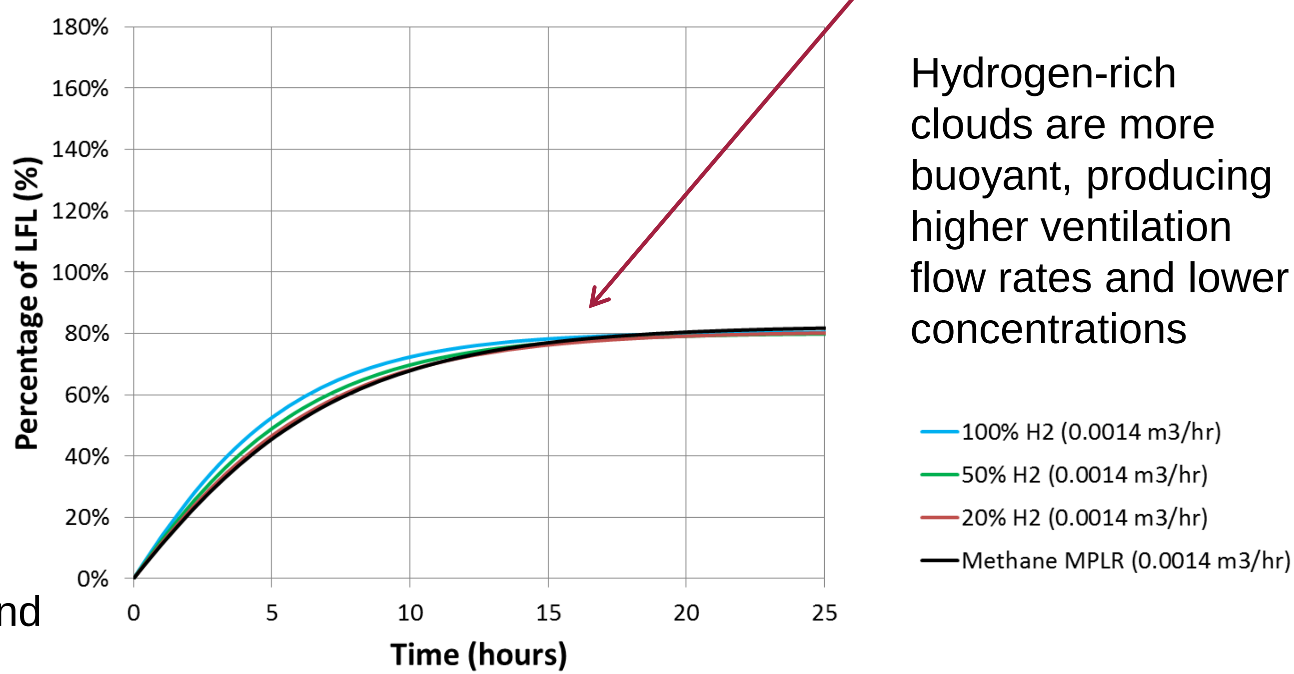
Gas Accumulation Results (Scenario 3)

Flow rates all set to 0.0014 m^3/hr (the current MPLR for natural gas)

No increased risk of producing flammable cloud



0.05 mm wide opening top and bottom



Hydrogen-rich clouds are more buoyant, producing higher ventilation flow rates and lower concentrations

Gas leak in metering cupboard

Is the solution to make the MPLR for hydrogen blends equal to the MPLR for natural gas on a volumetric (not energy) basis, equal to 0.0014 m^3/hr?

Conclusions

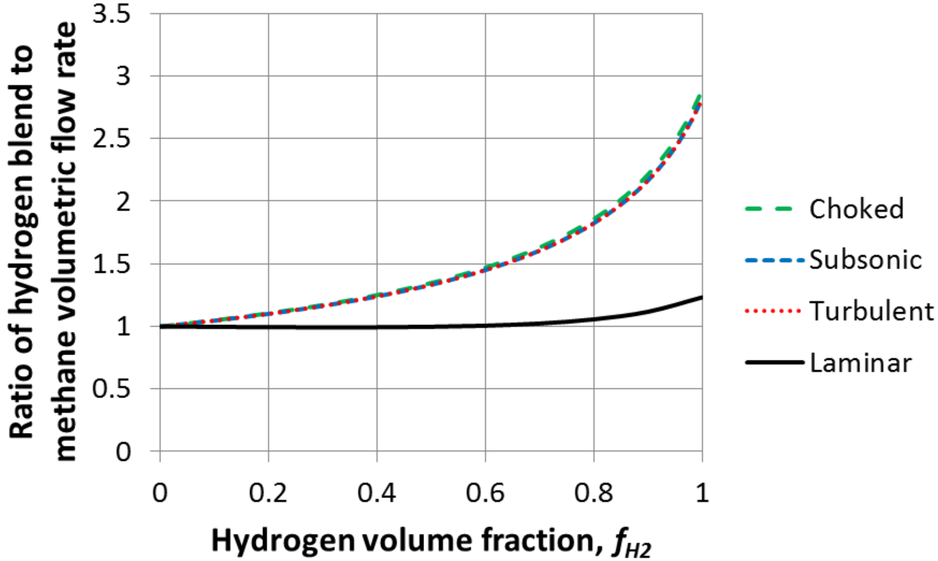
The aim of this work was address the following questions:

- Does hydrogen leak more than natural gas?
  - If so, by how much?
- What is its effect on the size of the flammable cloud?
- What are the implications for procedures, like IGE/UP/1?

Conclusions

Does hydrogen leak more than natural gas?

- Laminar leaks:
  - Blends < 70% v/v hydrogen, no change in volume flow rate
  - Blends > 70% v/v hydrogen, increase up to factor of 1.23 for pure hydrogen
- Turbulent/choked/subsonic leaks:
  - Increase in volume flow rate up to factor of 2.8 for pure hydrogen



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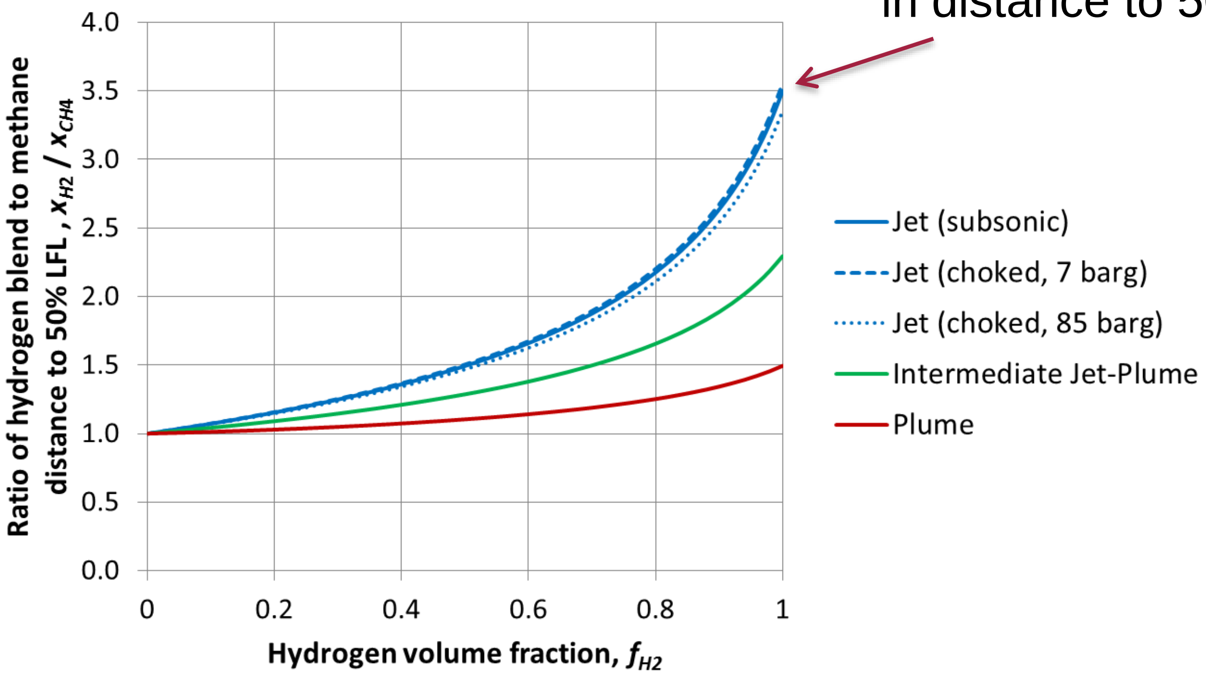
Conclusions

What is its effect on the size of the flammable cloud?

- For free vertical releases, an increased size of flammable cloud relative to natural gas

Factor of 3.5 increase in distance to 50% LFL



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Conclusions

What are the implications for procedures, like IGE/UP/1?

- For gas installations tested to IGE/UP/1 with natural gas, the gas accumulation model gave no increased risk of forming a flammable cloud from adding 20% or 50% hydrogen into natural gas
- If the MPLR for hydrogen is defined using an energy flow rate of 0.054 MJ/hr there is an increased risk of producing flammable clouds
- If the MPLR for hydrogen is defined as the same volume flow rate as natural gas (0.0014 m^3/hr), there is no increased risk of producing a flammable cloud (including blends with up to 100% hydrogen)

GAS TYPE	MPLR ($m^3 h^{-1}$) at OP
NATURAL	0.0014
BUTANE	0.00044
PROPANE	0.00057
LPG/AIR (SNG)	0.0013
LPG/AIR (SMG)	0.0021
COAL GAS	0.0029

TABLE 7 - MPLR (NEW INSTALLATIONS AND EXTENSIONS)

Solution:
Hydrogen 0.0014 m^3/hr?

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Acknowledgements

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Thank you

Any questions?

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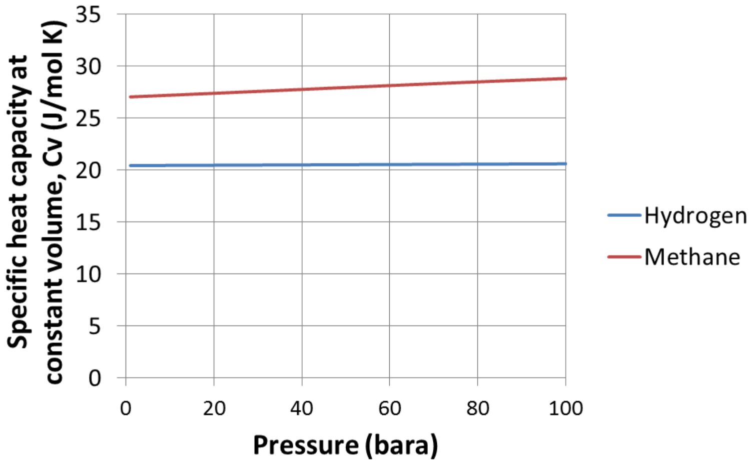
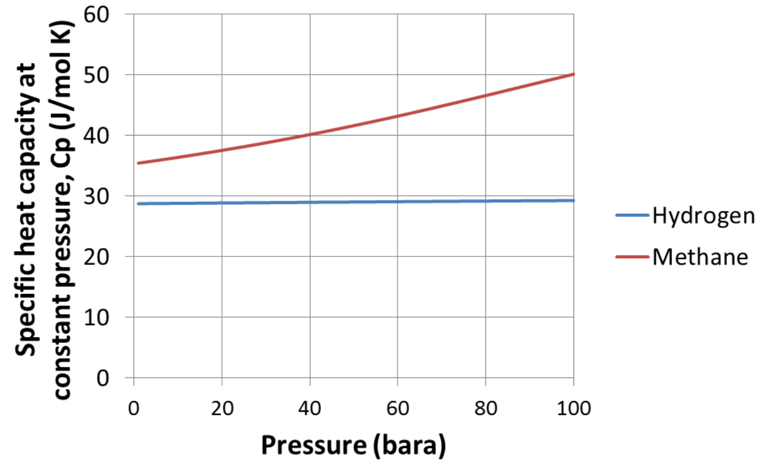
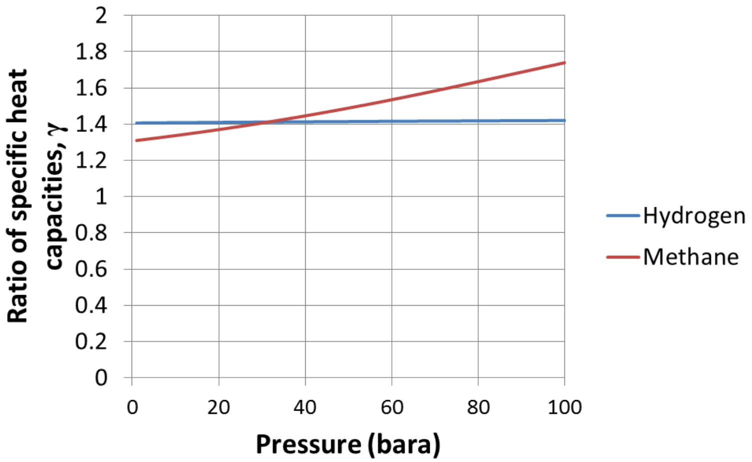
Ratio of specific heat capacities







Data at 15 °C from NIST webbook
https://webbook.nist.gov

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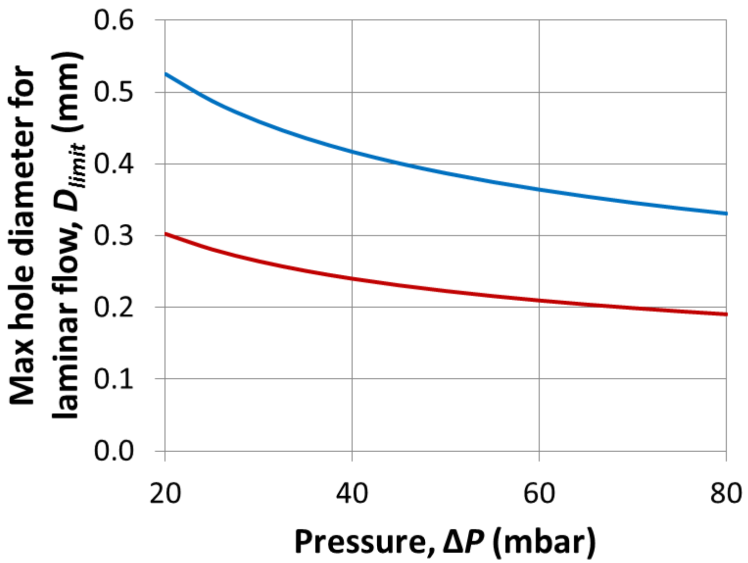
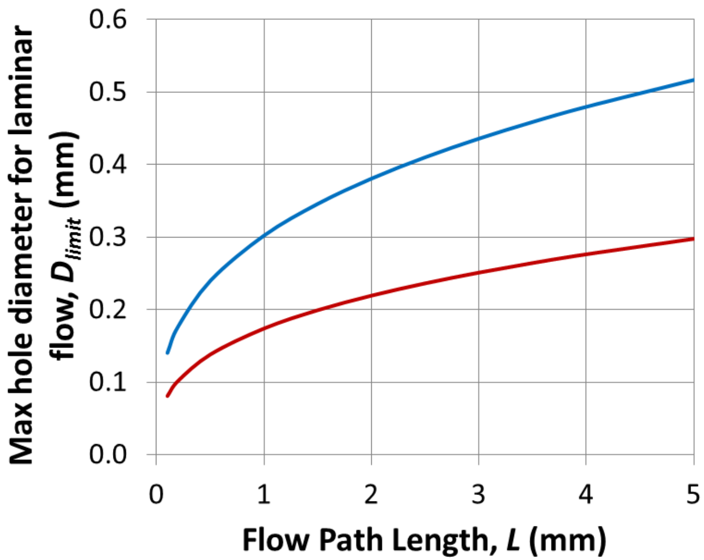
When is the flow laminar or turbulent?



Assuming leak flow path length of $L = 5$ mm



Assuming pressure of $P = 21$ mbarg

Maximum limiting hole diameters for laminar flow: hydrogen, methane

Conclusion: hydrogen more likely to produce laminar flow than methane

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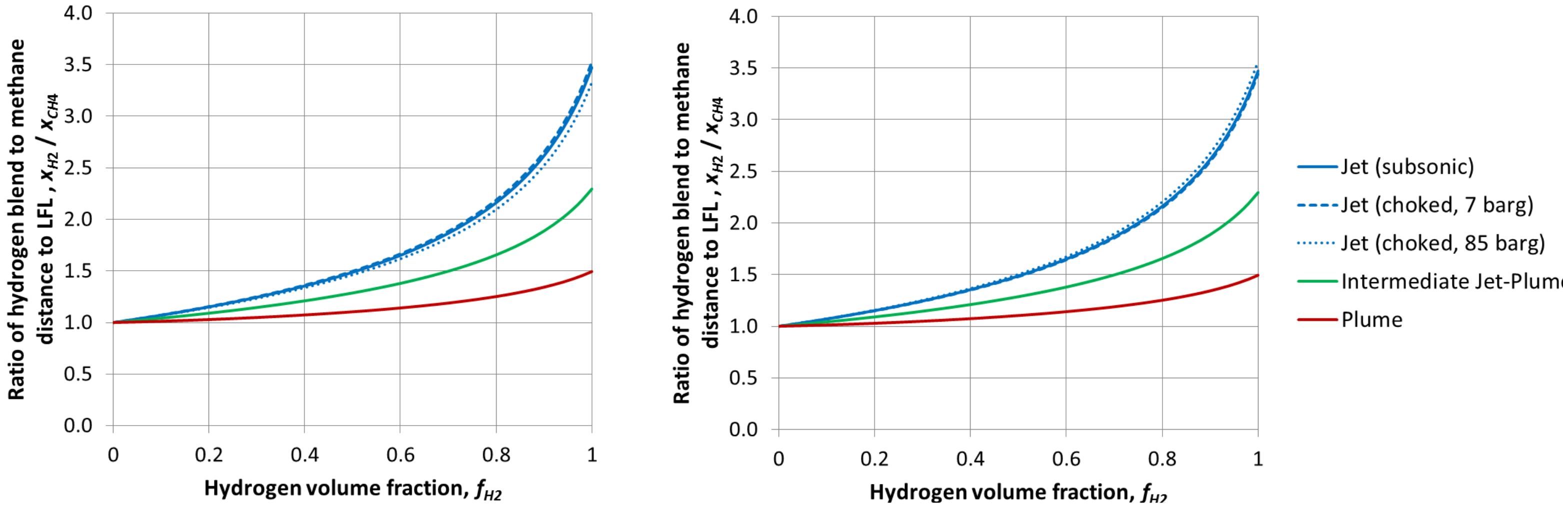
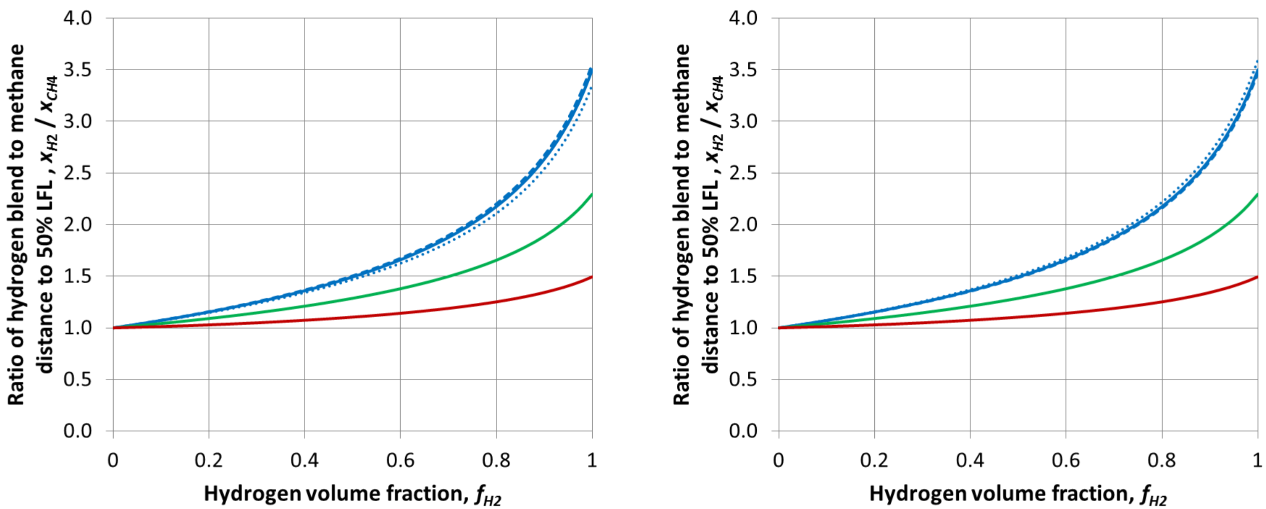
Ratio of distance to 100% LFL and 50% LFL, using Birch *et al*. (1987) and Ewan & Moodie (1986) models for choked releases

Birch *et al*. (1987) | Ewan & Moodie (1986)

100% LFL



50% LFL



Conclusion: All four results practically identical

IGE/UP/1

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5.5.2.6 *Test criteria*

The test criteria given in Table 8 shall be applied.

GAS TYPE	MPLR (m^3 h^{-1} (st)) LOCATION OF PIPEWORK: Inadequately ventilated areas (Area type A)	Adequately ventilated internal area volume ≤ 60m^3 rate per m^3 of smallest space volume (Area type B)	Adequately ventilated internal area. Volume 60m^3 or greater, external exposed, or buried (Area type C and D)
NATURAL	0.0014	0.0005	0.03
BUTANE	0.00044	0.00016	0.0098
PROPANE	0.00057	0.0002	0.0123
LPG/AIR(SNG)	0.0013	0.00046	0.0277
LPG/AIR(SMG)	0.0021	0.00075	0.045
COAL GAS	0.0029	0.001	0.062

TABLE 8 - MPLR (EXISTING INSTALLATIONS) ASSUMING TTP = OP

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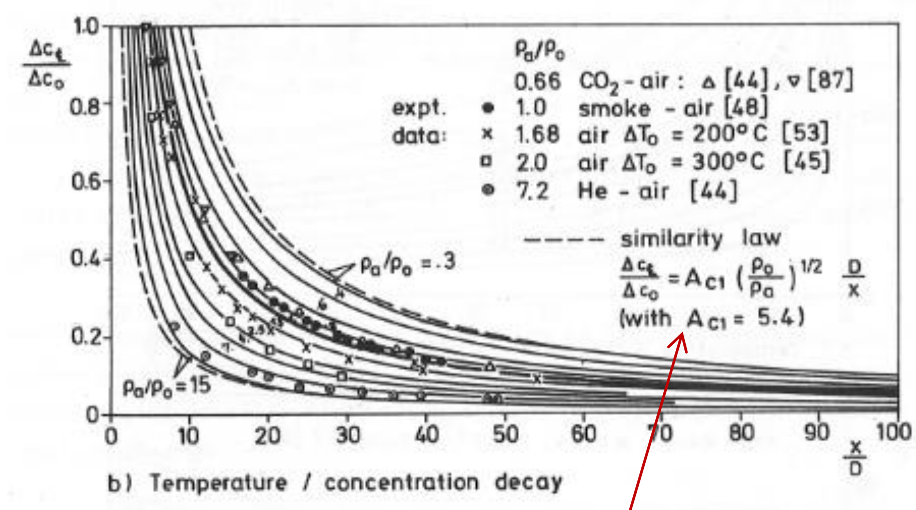
Jet Correlation

Concentration (mass fraction)

$$y = 5.4\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D}{x}$$

Rearranged in terms of distance

$$x = 5.4\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D}{y}$$



Original as it appears in Chen and Rodi (1980)

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{\rho_{H2}}{\rho_{CH4}}\right)^{\frac{1}{2}}\frac{y_{CH4}}{y_{H2}} = \left(\frac{M_{H2}}{M_{CH4}}\right)^{\frac{1}{2}}\frac{y_{CH4}}{y_{H2}}$$

Jet Correlation

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{\rho_{H2}}{\rho_{CH4}}\right)^{\frac{1}{2}} \frac{y_{CH4}}{y_{H2}} = \left(\frac{M_{H2}}{M_{CH4}}\right)^{\frac{1}{2}} \frac{y_{CH4}}{y_{H2}}$$

Mass fraction *y* as function of volume fraction *f*

$$y = \frac{f M_{gas}}{f M_{gas} + (1 - f) M_{air}}$$

Using LFL_{CH4} = 5.0 % v/v

$$y_{CH4} = \frac{(0.05)(16.043)}{(0.05)(16.043) + (1 - 0.05)28.97} = 0.028$$

Using LFL_{CH4} = 4.4 % v/v

$$y_{CH4} = \frac{(0.044)(16.043)}{(0.044)(16.043) + (1 - 0.044)28.97} = 0.025$$

Using LFL_{H2} = 4.0 % v/v

$$y_{H2} = \frac{(0.040)(2.016)}{(0.040)(2.016) + (1 - 0.040)28.97} = 0.0029$$

Jet Correlation

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{\rho_{H2}}{\rho_{CH4}}\right)^{\frac{1}{2}} \frac{y_{CH4}}{y_{H2}} = \left(\frac{M_{H2}}{M_{CH4}}\right)^{\frac{1}{2}} \frac{y_{CH4}}{y_{H2}}$$

Using LFL_{CH4} = 5.0 % v/v
(in mass terms 2.8 % w/w)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{2}{16}\right)^{\frac{1}{2}} \frac{2.8}{0.29} = 3.5$$

Using LFL_{CH4} = 4.4 % v/v
(in mass terms 2.5 % w/w)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{2}{16}\right)^{\frac{1}{2}} \frac{2.5}{0.29} = 3.0$$

Jet Correlation: Choked

Where did these formulae come from for the flammable cloud size for choked releases?

Birch *et al*. (1987)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{M_{H2}}{M_{CH4}}\right)^{\frac{1}{2}} \frac{y_{CH4}}{y_{H2}} \frac{f(\gamma_{H2})}{f(\gamma_{CH4})} = \left(\frac{2}{16}\right)^{\frac{1}{2}} \frac{2.8}{0.29} 1.02 = 3.5$$

Ewan & Moodie (1986)

$$\frac{x_{H2}}{x_{CH4}} = \left[\frac{M_{H2}}{M_{CH4}} \frac{(\gamma_{H2}+1)}{(\gamma_{CH4}+1)}\right]^{\frac{1}{2}} \frac{y_{CH4}}{y_{H2}} \left(\frac{2}{\gamma_{H2}+1}\right)^{\frac{\gamma_{H2}}{\gamma_{H2}-1}} \left(\frac{2}{\gamma_{CH4}+1}\right)^{-\frac{\gamma_{CH4}}{\gamma_{CH4}-1}} = 3.4$$

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Jet Correlation: Choked (Birch *et al*., 1987)

Concentration (mass fraction)

$$y = k\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D_{eff}}{x+a}$$

Concentration offset distance, *a*, is small and is ignored

Velocity ratio

$$\frac{V_3}{V_2} = C_d + \frac{1}{\gamma C_d}\left[1 - \frac{P_{atm}}{P_1}\left(\frac{2}{\gamma+1}\right)^{-\frac{\gamma}{\gamma-1}}\right]$$

$$\frac{d_e}{d} + \sqrt{\left[C_D\frac{P_1}{P_a}\left(\frac{2}{\gamma+1}\right)^{1/\gamma-1}\frac{V_2}{V_3}\right]},$$

$$V_3 = V_2\left\{C_D + \left[1 - \frac{P_a}{P_1}\left(\frac{2}{\gamma+1}\right)^{-\gamma/\gamma-1}\right]/\gamma C_D\right\}$$

Critical pressure

$$P_c = P_{atm}\left(\frac{2}{\gamma+1}\right)^{-\gamma/(\gamma-1)}$$

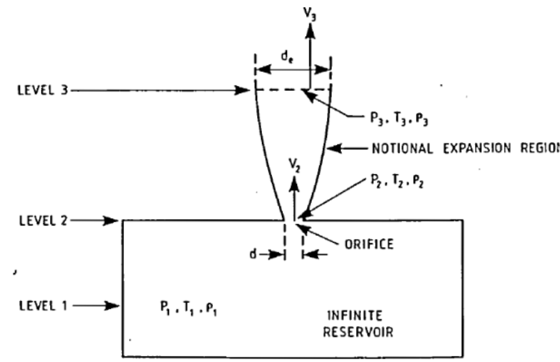


FIGURE 1 Super-critical gas release.

Effective source diameter

$$\frac{D_{eff}}{D} = \left[C_d\frac{P}{P_a}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left\{C_d + \frac{1}{\gamma C_d}\left[1 - \frac{P_c}{P}\right]\right\}^{-1}\right]^{\frac{1}{2}}$$

RESEARCH AND GUIDANCE FROM HSE

Jet Correlation: Choked (Birch *et al*., 1987)

Effective source diameter

$$\frac{D_{eff}}{D} = \left[C_d \frac{P}{P_a} \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \left\{ C_d + \frac{1}{\gamma C_d} \left[1 - \frac{P_c}{P} \right] \right\}^{-1} \right]^{\frac{1}{2}}$$

For pressures much higher than the critical pressure $(P \gg P_c)$

$$\frac{D_{eff}}{D} = \left[C_d \frac{P}{P_a} \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \left\{ C_d + \frac{1}{\gamma C_d} [1 - 0] \right\}^{-1} \right]^{\frac{1}{2}}$$

$$\frac{D_{eff}}{D} = \left[C_d \frac{P}{P_a} \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \left\{ \frac{\gamma C_d^2 + 1}{\gamma C_d} \right\}^{-1} \right]^{\frac{1}{2}}$$

$$\frac{D_{eff}}{D} = C_d \left[\frac{P}{P_a} \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \frac{\gamma}{(\gamma C_d^2 + 1)} \right]^{\frac{1}{2}}$$

Missing γ in Birch *et al*. (1987) paper

$$\frac{d_e}{d} = C_D \sqrt{\left[\frac{P_1}{P_a} \left(\frac{2}{\gamma + 1} \right)^{1/\gamma - 1} \frac{1}{(\gamma C_D^2 + 1)} \right]}.$$

RESEARCH AND GUIDANCE FROM HSE

Jet Correlation: Choked (Birch *et al*., 1987)

Concentration (mass fraction)

$$y = k\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D_{eff}}{x} \Rightarrow x = k\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D_{eff}}{y}$$

Effective source diameter

$$\frac{D_{eff}}{D} = C_d\left[\frac{P}{P_a}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\frac{\gamma}{(\gamma C_d^2+1)}\right]^{\frac{1}{2}}$$

$$\frac{D_{eff}}{D} = C_d\left(\frac{P}{P_a}\right)^{\frac{1}{2}}\underbrace{\left[\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\frac{\gamma}{(\gamma C_d^2+1)}\right]^{\frac{1}{2}}}_{f(\gamma)}$$

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{M_{H2}}{M_{CH4}}\right)^{\frac{1}{2}}\frac{y_{CH4}}{y_{H2}}\frac{f(\gamma_{H2})}{f(\gamma_{CH4})} = \left(\frac{2}{16}\right)^{\frac{1}{2}}\frac{2.8}{0.29}1.02 = 3.5$$

RESEARCH AND GUIDANCE FROM HSE

Jet Correlation: Choked (Ewan & Moodie, 1986)

Concentration (mass fraction)

$$y = k\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\frac{D_{eff}}{x+a}$$

Concentration offset distance, *a*, is small and is ignored

Effective source diameter

$$D_{eff} = D\left(\frac{P_e}{P_a}\right)^{\frac{1}{2}}$$

Thus the equivalent jet diameter may be given by

$$D_{eq} = D_j\left(\frac{P_e}{P_a}\right)^{0.5}.$$

Exit pressure

$$P_e = P\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_e = P_0\left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1},$$

Source density

$$\rho_0 = \rho_g\left(\frac{2}{\gamma+1}\right)^{-1}$$

$$\rho_{eq} = \rho_g\left(\frac{2}{\gamma+1}\right)^{-1},$$

Combined:

$$x = k\left[\frac{\rho_g}{\rho_a}\frac{(\gamma+1)}{2}\right]^{\frac{1}{2}}\frac{D}{y}\frac{P}{P_a}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

RESEARCH AND GUIDANCE FROM HSE

Jet Correlation: Choked (Ewan & Moodie, 1986)

$$x = k\left[\frac{\rho_g}{\rho_a}\frac{(\gamma+1)}{2}\right]^{\frac{1}{2}}\frac{D}{y}\frac{P}{P_a}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left[\frac{M_{H2}}{M_{CH4}}\frac{(\gamma_{H2}+1)}{(\gamma_{CH4}+1)}\right]^{\frac{1}{2}}\frac{y_{CH4}}{y_{H2}}\left(\frac{2}{\gamma_{H2}+1}\right)^{\frac{\gamma_{H2}}{\gamma_{H2}-1}}\left(\frac{2}{\gamma_{CH4}+1}\right)^{-\frac{\gamma_{CH4}}{\gamma_{CH4}-1}}$$

RESEARCH AND GUIDANCE FROM HSE

Plume Correlation

Where did this formula come from for the flammable cloud size for plumes?

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{C^*_{CH4}}{C^*_{H2}}\right)^{\frac{3}{5}} \left(\frac{U_{H2}}{U_{CH4}}\right)^{\frac{2}{5}} \left(\frac{M_{air} - M_{CH4}}{M_{air} - M_{H2}}\right)^{\frac{1}{5}} = \left(\frac{5.0}{4.0}\right)^{\frac{3}{5}} (2.8)^{\frac{2}{5}} \left(\frac{29-16}{29-2}\right)^{\frac{1}{5}} = 1.5$$

Buoyant Plume Correlation

Concentration (volume fraction)

$$C^* = 9.35 Fr^{\frac{1}{3}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{3}} \left(\frac{x}{D}\right)^{-\frac{5}{3}}$$

$$C^* = 9.35\ F^{1/3}\left(\frac{\rho_o}{\rho_a}\right)^{-1/3}\left(\frac{x}{D}\right)^{-5/3}$$

Original as it appears in Chen and Rodi (1980)

Froude number

$$Fr = \frac{U_0^2}{gD\,(\rho_a - \rho_0)/\rho_0}$$

Combined:

$$\left(\frac{x}{D}\right)^{\frac{5}{3}} = \frac{9.35}{C^*}\left(\frac{U_0^2}{gD\,(\rho_a - \rho_0)/\rho_0}\right)^{\frac{1}{3}}\left(\frac{\rho_a}{\rho_0}\right)^{\frac{1}{3}}$$

$$x = D\left(\frac{9.35}{C^*}\right)^{\frac{3}{5}}\left(\frac{\rho_a U_0^2}{gD(\rho_a - \rho_0)}\right)^{\frac{1}{5}}$$

Buoyant Plume Correlation

$$x = D\left(\frac{9.35}{C^*}\right)^{\frac{3}{5}}\left(\frac{\rho_a U_0^2}{gD(\rho_a - \rho_0)}\right)^{\frac{1}{5}}$$

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \frac{D\left(\frac{9.35}{C^*_{H2}}\right)^{\frac{3}{5}}\left(\frac{\rho_a U_{H2}^2}{gD(\rho_a - \rho_{H2})}\right)^{\frac{1}{5}}}{D\left(\frac{9.35}{C^*_{CH4}}\right)^{\frac{3}{5}}\left(\frac{\rho_a U_{CH4}^2}{gD(\rho_a - \rho_{CH4})}\right)^{\frac{1}{5}}}$$

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{C^*_{CH4}}{C^*_{H2}}\right)^{\frac{3}{5}}\left(\frac{\rho_a - \rho_{CH4}}{\rho_a - \rho_{H2}}\right)^{\frac{1}{5}}\left(\frac{U_{H2}}{U_{CH4}}\right)^{\frac{2}{5}}$$

Buoyant Plume Correlation

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{C^*_{CH4}}{C^*_{H2}}\right)^{\frac{3}{5}} \left(\frac{\rho_a - \rho_{CH4}}{\rho_a - \rho_{H2}}\right)^{\frac{1}{5}} \left(\frac{U_{H2}}{U_{CH4}}\right)^{\frac{2}{5}}$$

Inserting values
(LFL_{CH4} = 5.0 % v/v)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{0.05}{0.04}\right)^{\frac{3}{5}} \left(\frac{28.97 - 16.043}{28.97 - 2.016}\right)^{\frac{1}{5}} (2.82)^{\frac{2}{5}} = 1.5$$

Inserting values
(LFL_{CH4} = 4.4 % v/v)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{0.044}{0.040}\right)^{\frac{3}{5}} \left(\frac{28.97 - 16.043}{28.97 - 2.016}\right)^{\frac{1}{5}} (2.82)^{\frac{2}{5}} = 1.4$$

RESEARCH AND GUIDANCE FROM HSE

Intermediate Jet-Plume Correlation

Where did this formula come from for the flammable cloud size for intermediate jet-plumes?

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{C^*_{CH4}}{C^*_{H2}}\right)^{\frac{4}{5}} \left(\frac{U_{H2}}{U_{CH4}}\right)^{\frac{1}{5}} \left(\frac{M_{air} - M_{CH4}}{M_{air} - M_{H2}}\right)^{\frac{1}{10}} \left(\frac{M_{CH4}}{M_{H2}}\right)^{\frac{1}{4}} = \left(\frac{0.05}{0.04}\right)^{\frac{4}{5}} (2.8)^{\frac{1}{5}} \left(\frac{29-16}{29-2}\right)^{\frac{1}{10}} \left(\frac{16}{2}\right)^{\frac{1}{4}} = 2.3$$

RESEARCH AND GUIDANCE FROM HSE

Intermediate Jet-Plume Correlation

Concentration (volume fraction)

$$C^* = 4.4 Fr^{\frac{1}{8}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{7}{16}} \left(\frac{x}{D}\right)^{-\frac{5}{4}}$$

$$C^* = 0.44\, F^{1/8} \left(\frac{\rho_o}{\rho_a}\right)^{-7/16} \left(\frac{x}{D}\right)^{-5/4}$$

should be 4.4?

Original as it appears in Chen and Rodi (1980)

Froude number

$$Fr = \frac{U_0^2}{gD\,(\rho_a - \rho_0)/\rho_0}$$

Combined:

$$\left(\frac{x}{D}\right)^{\frac{5}{4}} = \frac{4.4}{C^*} \left(\frac{U_0^2}{gD\,(\rho_a - \rho_0)/\rho_0}\right)^{\frac{1}{8}} \left(\frac{\rho_a}{\rho_0}\right)^{\frac{7}{16}}$$

$$x = D \left(\frac{4.4}{C^*}\right)^{\frac{4}{5}} \left(\frac{U_0^2}{gD\,(\rho_a - \rho_0)/\rho_0}\right)^{\frac{1}{10}} \left(\frac{\rho_a}{\rho_0}\right)^{\frac{7}{20}}$$

$$x = D \left(\frac{4.4}{C^*}\right)^{\frac{4}{5}} U_0^{\frac{1}{5}} \left(\frac{1}{gD}\right)^{\frac{1}{10}} \left(\frac{1}{(\rho_a - \rho_0)}\right)^{\frac{1}{10}} (\rho_a)^{\frac{7}{20}} \left(\frac{1}{\rho_0}\right)^{\frac{1}{4}}$$

Intermediate Jet-Plume Correlation

$$x = D\left(\frac{4.4}{C^*}\right)^{\frac{4}{5}} U_0^{\frac{1}{5}} \left(\frac{1}{gD}\right)^{\frac{1}{10}} \left(\frac{1}{(\rho_a - \rho_0)}\right)^{\frac{1}{10}} (\rho_a)^{\frac{7}{20}} \left(\frac{1}{\rho_0}\right)^{\frac{1}{4}}$$

Ratio of hydrogen to methane distance to LFL

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{C^*_{CH4}}{C^*_{H2}}\right)^{\frac{4}{5}} \left(\frac{U_{H2}}{U_{CH4}}\right)^{\frac{1}{5}} \left(\frac{\rho_a - \rho_{CH4}}{\rho_a - \rho_{H2}}\right)^{\frac{1}{10}} \left(\frac{\rho_{CH4}}{\rho_{H2}}\right)^{\frac{1}{4}}$$

Inserting values (LFL_{CH4} = 5.0 % v/v)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{0.05}{0.04}\right)^{\frac{4}{5}} (2.82)^{\frac{1}{5}} \left(\frac{28.97 - 16.043}{28.97 - 2.016}\right)^{\frac{1}{10}} \left(\frac{16.043}{2.016}\right)^{\frac{1}{4}} = 2.3$$

Inserting values (LFL_{CH4} = 4.4 % v/v)

$$\frac{x_{H2}}{x_{CH4}} = \left(\frac{0.044}{0.040}\right)^{\frac{4}{5}} (2.82)^{\frac{1}{5}} \left(\frac{28.97 - 16.043}{28.97 - 2.016}\right)^{\frac{1}{10}} \left(\frac{16.043}{2.016}\right)^{\frac{1}{4}} = 2.1$$

When are releases jets or plumes?

Transition from jet to plume is controlled by parameter, *B*

$$B = Fr^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{x}{D}$$

$$0.5 \leq F^{-1/2} \left(\frac{\rho_o}{\rho_a}\right)^{-1/4} \frac{x}{D} \leq 5$$

$B < 0.5$	the flow is a momentum-dominated jet
$0.5 < B < 5.0$	the flow is in an intermediate state between jet and plume
$B > 5.0$	the flow is a buoyancy-dominated plume

Rearrange in terms of distance:

$$\frac{D}{x} = Fr^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{1}{B}$$

Substitute in jet formula:

$$y = 5.4 \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}} \frac{D}{x}$$

To give:

$$y = 5.4 \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}} \left[\frac{\rho_0 U_0^2}{gD(\rho_a - \rho_0)}\right]^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{1}{B}$$

When are releases jets or plumes?

$$y = 5.4\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}}\left[\frac{\rho_0 U_0^2}{gD(\rho_a - \rho_0)}\right]^{-\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}}\frac{1}{B}$$

Rearrange in terms of release velocity:

$$U_0 = \frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$

Equation for subsonic release velocity from BS EN 60079-10-1:

$$U_0 = \frac{\dot{m}}{\rho_0 A} = \frac{C_d P}{\rho_0}\sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma - 1)}\left[1 - \left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$$

Equate the two equations for the release velocity:

$$\frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}} = \frac{C_d P}{\rho_0}\sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma - 1)}\left[1 - \left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$$

RESEARCH AND GUIDANCE FROM HSE

When are releases jets or plumes?

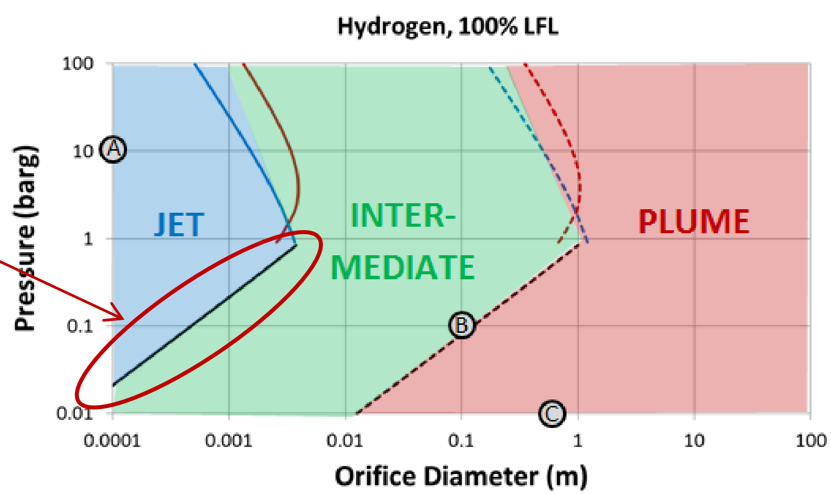
$$\frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD(\rho_a-\rho_0)}{\rho_0}\right]^{\frac{1}{2}}=\frac{C_dP}{\rho_0}\sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$$

Rearrange for the release diameter as a function of pressure:

$$D=\left[\frac{\rho_0}{g(\rho_a-\rho_0)}\right]\left(\frac{yB}{5.4}\right)^2\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{2}}\left(\frac{C_dP}{\rho_0}\right)^2\left\{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]\right\}\left(\frac{P_{atm}}{P}\right)^{2/\gamma}$$

This equation is used to draw the line for subsonic releases at the boundary between jet and intermediate

$B = 0.5$ at boundary



When are releases jets or plumes?

Now for choked releases, using the Birch *et al*. (1987) pseudo-source model

$$B = Fr^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{x}{D_{eff}}$$

Rearrange in terms of distance:

$$\frac{D_{eff}}{x} = Fr^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{1}{B}$$

Substitute in choked jet formula:

$$y = 5.4 \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}} \frac{D_{eff}}{x}$$

To give:

$$y = 5.4 \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}} \left[\frac{\rho_0 U_0^2}{g D_{eff}(\rho_a - \rho_0)}\right]^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{1}{B}$$

Rearrange in terms of release velocity:

$$U_0 = \frac{5.4}{yB} \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}} \left[\frac{g D_{eff}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$

When are releases jets or plumes?

$$U_0 = \frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD_{eff}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$

Equation for expanded jet velocity from BS EN 60079-10-1:

$$U_0 = \frac{\dot{m}}{\rho_0 A_{eff}} = \frac{C_d P}{\rho_0}\frac{A}{A_{eff}}\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

Birch *et al*. (1987) effective source diameter:

$$\frac{D_{eff}}{D} = C_{eff} = \left[C_d\frac{P}{P_a}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left\{C_d + \frac{1}{\gamma C_d}\left[1 - \frac{P_c}{P}\right]\right\}^{-1}\right]^{\frac{1}{2}}$$

where:

$$\frac{A}{A_{eff}} = \frac{\pi D^2/4}{\pi D_{eff}^2/4} = \frac{1}{C_{eff}^2}$$

Equating the velocities:

$$\frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD_{eff}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}} = \frac{C_d P}{\rho_0 C_{eff}^2}\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

When are releases jets or plumes?

$$\frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD_{eff}(\rho_a-\rho_0)}{\rho_0}\right]^{\frac{1}{2}} = \frac{C_d P}{\rho_0 C_{eff}^2}\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

Rearranging the equation into orifice diameter and a function of pressure:

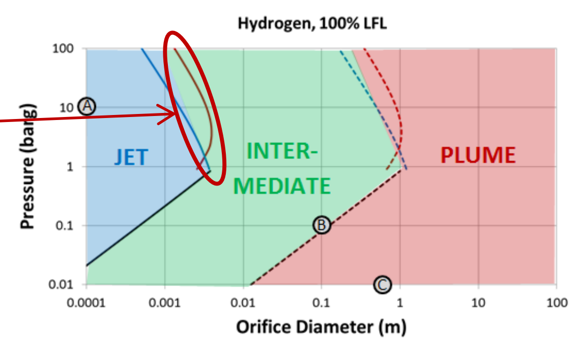
$$D = \frac{1}{C_{eff}}\left[\frac{\rho_0}{g(\rho_a-\rho_0)}\right]\left(\frac{yB}{5.4}\right)^2\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{2}}\left(\frac{C_d P}{\rho_0 C_{eff}^2}\right)^2\left[\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}\right]$$

where:

$$C_{eff} = \left[C_d\frac{P}{P_a}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left\{C_d+\frac{1}{\gamma C_d}\left[1-\frac{P_c}{P}\right]\right\}^{-1}\right]^{\frac{1}{2}}$$

This equation is used to draw the line for choked releases at the boundary between jet and intermediate

$B = 0.5$ at boundary



When are releases jets or plumes?

Now for choked releases using the Ewan & Moodie (1986) pseudo-source model

Same derivation as before to give the orifice diameter in terms of pressure

$$D = \frac{1}{C_{eff}}\left[\frac{\rho_0}{g(\rho_a - \rho_0)}\right]\left(\frac{yB}{4.99}\right)^2\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{2}}\left(\frac{C_d P}{\rho_0 C_{eff}^2}\right)^2\left[\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}\right]$$

Note that Ewan and Moodie use k = 4.99 instead of 5.4 in the concentration-decay formula

Ewan & Moodie (1986) effective source diameter: $\frac{D_{eff}}{D} = C_{eff} = \left(\frac{P_e}{P_{atm}}\right)^{\frac{1}{2}}$

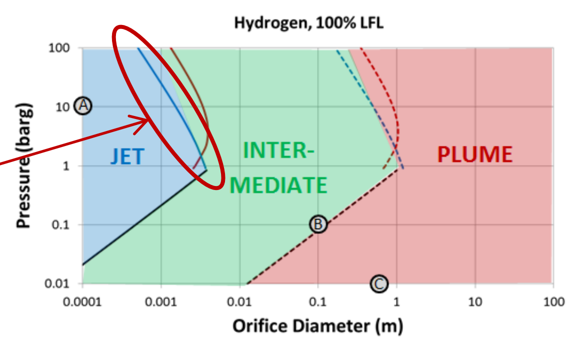
Exit pressure: $P_e = P\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

Pseudo-source density $\rho_0 = \rho_g\left(\frac{2}{\gamma+1}\right)^{-1}$

where P is the upstream stagnation pressure

The above equations are used to draw the line for choked releases at the boundary between jet and intermediate

$B = 0.5$ at boundary

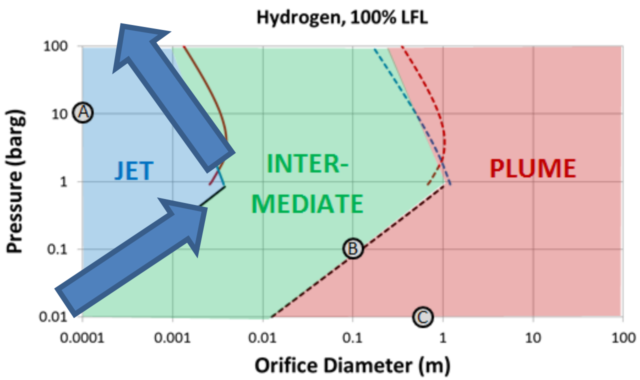


where ρ_g is the gas density at ambient temperature

RESEARCH AND GUIDANCE FROM HSE

When are releases jets or plumes?

Why does the trend change at the critical pressure from increasing D with P to decreasing D with P?



Below the critical pressure ($P < P_{crit}$), the boundary between jet and plume is defined by parameter B as follows:

$$B = \left[\frac{\rho_0 U_0^2}{gD(\rho_a - \rho_0)}\right]^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{x}{D}$$

$B < 0.5$ jet
$0.5 < B < 5.0$ intermediate
$B > 5.0$ plume

Substituting the concentration decay formula for jets:

$$y = 5.4 \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{2}} \frac{D_{eff}}{x}$$

The resulting equation rearranged is:

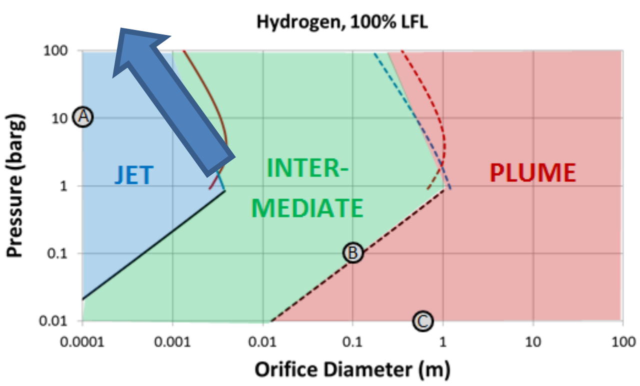
$$U_0 = \frac{5.4}{yB} \left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}} \left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$

So an increase in pressure, which causes an increase in velocity U_0 is balanced by an increase in diameter D and we have the positive slope to the line in the diagram

When are releases jets or plumes?

Above the critical pressure ($P > P_{crit}$), the diameter in parameter B becomes the pseudo-source diameter, D_{eff}:

$$U_0 = \frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD_{eff}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$



Diameter D_{eff} is a function of pressure. In the Ewan & Moodie (1986) model $D_{eff} \propto DP^{\frac{1}{2}}$

$$U_0 = c\,\frac{5.4}{yB}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gDP^{\frac{1}{2}}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$

The flow is choked and so U_0 is not a function of pressure . Rearranging the above equation:

$$\frac{1}{P^{\frac{1}{4}}} = c\,\frac{5.4}{yBU_0}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{1}{4}}\left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}$$

So an increase in pressure P is balanced by a decrease in diameter D, hence the negative slope to the line in the diagram

When are releases jets or plumes?

Is the velocity U_0 really independent of pressure when the flow is choked?

Equation for expanded jet velocity from BS EN 60079-10-1:

$$U_0 = \frac{\dot{m}}{\rho_0 A_{eff}} = \frac{C_d P}{\rho_0} \frac{A}{A_{eff}} \sqrt{\gamma \frac{M}{ZRT} \left(\frac{2}{\gamma + 1} \right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

Ewan & Moodie (1986) pseudo-source diameter

$$\frac{D_{eff}}{D} = \left(\frac{P_e}{P_{atm}} \right)^{\frac{1}{2}} = \left(\frac{P}{P_{atm}} \right)^{\frac{1}{2}} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$\frac{A}{A_{eff}} = \frac{\pi D^2/4}{\pi D_{eff}^2/4} = \left(\frac{D}{D_{eff}} \right)^2$$

So the area ratio A/A_{eff} is proportional to $1/P$, which cancels the pressure term in the equation for the pseudo-source velocity, U_0, i.e. the velocity U_0 is not a function of pressure

RESEARCH AND GUIDANCE FROM HSE

When are releases jets or plumes?

How is the plume boundary calculated?

Starting from the parameter B, which takes a value of 5.0 at the plume boundary

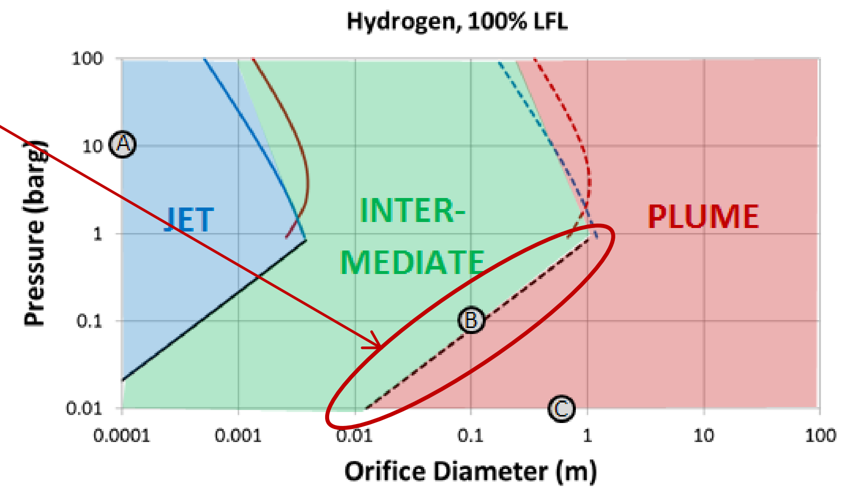
$$B = Fr^{-\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}}\frac{x}{D}$$

Chen & Rodi (1980) correlation for concentration decay in plumes:

$$C^* = 9.35Fr^{\frac{1}{3}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{3}}\left(\frac{x}{D}\right)^{-\frac{5}{3}}$$

Combining the two equations:

$$C^* = 9.35Fr^{\frac{1}{3}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{3}}\left[Fr^{-\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}}\frac{1}{B}\right]^{\frac{5}{3}}$$



When are releases jets or plumes?

$$C^* = 9.35 Fr^{\frac{1}{3}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{3}} \left[Fr^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{1}{4}} \frac{1}{B} \right]^{\frac{5}{3}}$$

Rearranging:

$$C^* = 9.35 Fr^{\frac{2}{6}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{4}{12}} Fr^{-\frac{5}{6}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{5}{12}} \left(\frac{1}{B}\right)^{\frac{5}{3}}$$

$$C^* = 9.35 \left[\frac{\rho_0 U_0^2}{gD(\rho_a - \rho_0)}\right]^{-\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}} \left(\frac{1}{B}\right)^{\frac{5}{3}}$$

$$U_0 = \frac{9.35}{C^*} \left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}} \left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}} \left(\frac{1}{B}\right)^{\frac{5}{3}}$$

When are releases jets or plumes?

$$U_0 = \frac{9.35}{C^*}\left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}}\left(\frac{1}{B}\right)^{\frac{5}{3}}$$

Equation for subsonic release velocity from BS EN 60079-10-1:

$$U_0 = \frac{\dot{m}}{\rho_0 A} = \frac{C_d P}{\rho_0}\sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$$

Equating these two equations for the velocity gives:

$$\frac{9.35}{C^*}\left[\frac{gD(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}}\left(\frac{1}{B}\right)^{\frac{5}{3}} = \frac{C_d P}{\rho_0}\sqrt{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]}\left(\frac{P_{atm}}{P}\right)^{1/\gamma}$$

And rearranging:

$$D = \left[\frac{\rho_0}{g(\rho_a - \rho_0)}\right]\left(\frac{C^*}{9.35}\right)^2\left(\frac{\rho_0}{\rho_a}\right)^{\frac{3}{2}}\left(\frac{1}{B}\right)^{-\frac{10}{3}}\left(\frac{C_d P}{\rho_0}\right)^2\left\{\frac{M}{ZRT}\frac{2\gamma}{(\gamma-1)}\left[1-\left(\frac{P_{atm}}{P}\right)^{(\gamma-1)/\gamma}\right]\right\}\left(\frac{P_{atm}}{P}\right)^{2/\gamma}$$

RESEARCH AND GUIDANCE FROM HSE

When are releases jets or plumes?

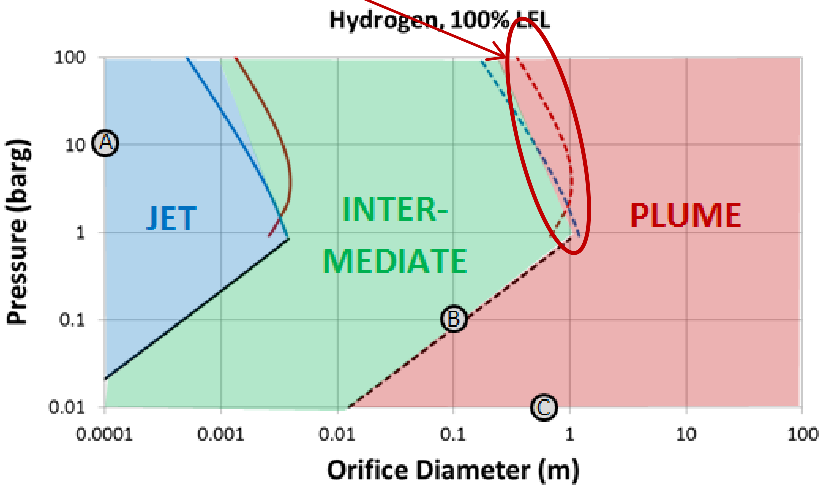
How is the plume boundary calculated when the flow is choked?

Starting from the equation for velocity presented previously for subsonic plumes, but using the pseudo-source diameter, D_{eff}:

$$U_0 = \frac{9.35}{C^*}\left[\frac{gD_{eff}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}}\left(\frac{1}{B}\right)^{\frac{5}{3}}$$

The equation for the choked flow velocity is:

$$U_0 = \frac{C_d P}{\rho_0}\frac{1}{C_{eff}^2}\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$



Combining these two equations for the velocity gives:

$$\frac{9.35}{C^*}\left[\frac{gD_{eff}(\rho_a - \rho_0)}{\rho_0}\right]^{\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}}\left(\frac{1}{B}\right)^{\frac{5}{3}} = \frac{C_d P}{\rho_0}\frac{1}{C_{eff}^2}\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

RESEARCH AND GUIDANCE FROM HSE

When are releases jets or plumes?

$$\frac{9.35}{C^*}\left[\frac{gD_{eff}(\rho_a-\rho_0)}{\rho_0}\right]^{\frac{1}{2}}\left(\frac{\rho_0}{\rho_a}\right)^{-\frac{3}{4}}\left(\frac{1}{B}\right)^{\frac{5}{3}}=\frac{C_dP}{\rho_0}\frac{1}{C_{eff}^2}\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

$$\left[\frac{gD_{eff}(\rho_a-\rho_0)}{\rho_0}\right]^{\frac{1}{2}}=\frac{C^*}{9.35}\left(\frac{\rho_0}{\rho_a}\right)^{\frac{3}{4}}\left(\frac{1}{B}\right)^{-\frac{5}{3}}\left(\frac{C_dP}{\rho_0}\frac{1}{C_{eff}^2}\right)\sqrt{\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

Rearranging using the relation $D_{eff}=C_{eff}D$:

$$D=\frac{1}{C_{eff}}\left[\frac{\rho_0}{g(\rho_a-\rho_0)}\right]\left(\frac{C^*}{9.35}\right)^2\left(\frac{\rho_0}{\rho_a}\right)^{\frac{3}{2}}\left(\frac{1}{B}\right)^{-\frac{10}{3}}\left(\frac{C_dP}{\rho_0}\frac{1}{C_{eff}^2}\right)^2\left[\gamma\frac{M}{ZRT}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}\right]$$

The equations presented previously from Birch *et al*. (1987) or Ewan & Moodie (1986) are then used for C_{eff} in the above expression