Chapter 7

Ahmed Body Flow

7.1 Introduction

The "Ahmed" body, shown in Figure 7.1, has the form of a highly simplified car, consisting of a blunt nose with rounded edges fixed onto a box-like middle section and a rear end that has an upper slanted surface (like a "hatch-back" car), the angle of which can be varied. The model is supported on circular-sectioned legs or stilts, rather than wheels. Despite neglecting a number of features of a real car (rotating wheels, rough underside, surface projections etc.) the Ahmed body generates the essential features of flow around a car, namely: flow impingement and displacement around the nose, relatively uniform flow around the middle and flow separation and wake generation at the rear.

The principal aim of studying such a simplified car body is to understand the flow processes involved in drag production. Through understanding the mechanisms involved in generating drag one should be able to design a car to minimize drag and therefore minimize fuel consumption and maximize performance.

The principal contribution to drag experienced by a car is pressure drag. The rear of the vehicle provides the major contribution to pressure drag and, in particular, the angle of the rear slant is critical in determining the mode of the wake flow and hence the drag experienced by the vehicle. Janssen & Hucho [135] found that the maximum drag was obtained for a vehicle with rear slant angle $\beta \approx 30°$ (to the horizontal) where the flow over the slant remained partially attached and longitudinal trailing vortices were formed at the edges of the slant. For steeper slant angles ($\beta > 30°$) the flow over the rear slant became fully-separated and the drag decreased.

7.2 Previous Experimental and Computational Studies

In the original experiments undertaken by Ahmed [136] and Ahmed *et al.* [137], the angle of the rear slant was varied from $\beta = 0°$ to $40°$. Visualization techniques were employed to examine the structure of the wake and time-averaged velocity measurements were made on the centreline plane and at transverse planes in the wake. Measurements of the total drag were made at 5° intervals for slant angles from $\beta = 0°$ to 40°. The total drag was observed to fall from $\beta = 0°$ to 15° and then rise to a maximum at 30°, followed by a sudden decrease, thereafter remaining almost constant between 30° and 40° (see Figure 7.2). A breakdown of the relative contributions to drag from pressure on the nose, base[1] and slant was taken at $\beta = 5°$, 12.5°, 30° (high drag) and 30° (low drag). A vertical splitter plate in the symmetry plane was used in the wake of the 30° slant to encourage the low-drag flow. This indicates that some unsteadiness in the wake may have been instrumental in maintaining the high-drag 30° slant flow.

A physical description of the flow structure over the rear of the Ahmed body was presented in some detail in the Ahmed papers and also in the more recent experiments undertaken by Lienhart *et al.* [138] and Spohn & Gilliéron [139]. Figures 7.3 and 7.4 show sketches of the time-averaged wake structure taken from the Ahmed *et al.* paper for the low drag $\beta \approx 20°$ and high drag $\beta = 30°$ configurations, Figure 7.5 shows oil/soot streakflow visualization of the 25° and 35° slants from Lienhart *et al.* and Figure 7.6 shows a diagrammatic representation of the wake over the 25° rear slant taken from Spohn and Gilliéron. The low-drag configuration of the Ahmed body, with a rear-slant angle of approximately 20°, is characterized by longitudinal vortices which originate along the edges between the side and the rear slant surfaces (vortex C in the Ahmed Figure 7.3). The formation of these side-edge vortices is similar to that observed for low aspect-ratio wings where the high pressure on the underside of the wing and low pressure above the wing forces fluid to flow around the edges of the wing-tips. In the case of the Ahmed body, the fast moving air around the side of the body is pulled into a low-pressure region just above the rear slant near the side edge and, in doing so, the flow curls over to create a vortex which has its axis roughly aligned to the slant edge. Tucked just underneath the large side-edge vortex is a smaller vortex which rotates in the opposite direction. The smaller side-edge vortex is shown clearly in the Spohn & Gilliéron diagram (Figure 7.6) and the attachment line which occurs on the surface of the slant between the large and the small side-edge vortices can be seen on the Ahmed diagram (Figure 7.3). The flow over the 20° slant is fully attached, promoting pressure recovery and low drag. Downstream of the base of the Ahmed body there are two horseshoe vortices (marked as A and B in Figure 7.3) which interact with the flow leaving the slant, the side-edge vortices and the flow from the underside of the body. As the slant angle is increased from 20° to 30° the strength of the side-edge vortices increases and a separation bubble appears at the leading edge of the slant (where the slant meets the top surface of the Ahmed body). On the 25° rear slant, the oil/soot streakflow shows a separated flow region near the leading edge which reattaches roughly half-way down the slant and remains attached until the edge is reached between the slant and the

[1]"base" refers to the vertical plane surface on the rear of the Ahmed body, below the slant.

vertical base. The separation bubble appears to be slightly larger in the diagram of Spohn & Gilliéron for the 25° slant which may be a consequence of the lower Reynolds number used in their experiments ($Re \approx 3 \times 10^4$, compared to the Lienhart *et al.* visualization which was carried out at $Re \approx 9 \times 10^4$) or the lower turbulence level in the boundary layer approaching the slant[2]. Spohn & Gilliéron also noted that whilst the flow near the leading edge of the slant is steady, the flow closer to the downstream end of the slant is highly unsteady. The low pressure induced by the separation bubble and the strong side-edge vortices lead to a high pressure-drag on the slant surface. The Ahmed figures show the separation bubble increasing in size as the slant angle is increased up to 30°. For the high-drag 30° slant, the bubble (shown as vortex E in Figure 7.4) reaches practically to the downstream edge of the rear slant before the boundary-layer reattaches.

For the low-drag configuration for the 30° slant, and at higher slant angles, the separation bubble over the rear slant merges with the upper of the two horseshoe vortices behind the base to create a large recirculation region with weaker side-edge vortices. This is indicated by the oil/soot visualization for the 35° rear slant (Figure 7.5).

A possible explanation for the switch from the high-drag mode at slant angles $20° < \beta < 30°$ to the low-drag mode at slant angles greater than 30° was put forward by Menter at the 2002 ERCOFTAC Workshop on Refined Turbulence Modelling[3]. It was suggested that the size of the high-pressure region (linked to the separation bubble located on the leading edge of the slant) grows as the slant angle is increased from 20° to 30°. At the critical slant angle of 30°, the high-pressure region is sufficiently large that it interferes with the low-pressure zone at the corner between the slant and the side of the Ahmed body. As the strength of this low-pressure zone is reduced so the main driving force behind the creation of the strong side-edge vortices is removed, the vortices become weaker and hence exert a reduced force on the flow over the slant which then becomes fully separated. Additionally, as the slant angle is increased up to the critical angle, the recirculation bubble on the rear slant extends beyond the downstream end of the slant and combines with the upper horseshoe vortex downstream of the base. As one tries to visualize this transition from high- to low-drag modes one should keep in mind that in reality the flow is highly unsteady. This was demonstrated clearly by the large-eddy simulation of flow around the 25° Ahmed body, presented by Hinterberger *et al.* at the 2002 ERCOFTAC workshop, which showed an unsteady wake over the rear slant

[2]Lienhart *et al.* employed a trip on the curved part of the front of the Ahmed body to increase turbulence levels in the boundary layer whereas Spohn & Gilliéron did not use a trip.

[3]Simulations of flow around the Ahmed body were presented at two ERCOFTAC Workshops on Refined Turbulence Modelling: at Darmstadt, Germany, in 2001, and at Poitiers, France, in 2002. Formal proceedings from the 2001 workshop have not been published although the details of the computational methods used by the participants and cross-plots of the data are available from the workshop coordinator, Dr Jakirlic, on the TU-Darmstadt website: http://www.sla.maschinenbau.tu-darmstadt.de/workshop01.html. At the time of writing, proceedings from the 2002 workshop have also not yet been published formally although it is planned to make the results freely available. Details of the 2002 meeting in Poitiers can be obtained from the workshop coordinator: Dr Manceau, Laboratoire d'études aérodynamiques, University of Poitiers, France.

with eddies that were occasionally becoming detached from the slant surface being convected downstream.

Perhaps the earliest simulation of the Ahmed body flow was undertaken by Han [140] who studied a range of different slant angles using a finite-volume code and standard linear $k-\varepsilon$ model with wall functions. Han found that the drag coefficient for slant angles in the range $0° < \beta < 20°$ were consistently predicted around 30% too high. This was attributed to an overly low pressure on the base of the body. At the critical slant angle of $\beta = 30°$ no separation was predicted over the slant and the steep rise in drag reported by Ahmed for slant angles in the range $25° < \beta < 30°$ was not calculated. Wilcox [14] suggested that the poor predictions obtained by Han were due to the linear $k-\varepsilon$ model which is known to perform poorly in flows involving strong adverse pressure gradients. It was suggested that the overly low base pressure was caused by a vortex which was too strong, which was itself the result of an overly large wall shear stress being predicted by the $k-\varepsilon$ model. Robinson [35] noted that there are a number of other considerations which may have influenced Han's results. The linear $k-\varepsilon$ model is well known to overpredict the turbulent kinetic energy at a stagnation point and this may have led to an overly large level of turbulence energy in the boundary layer approaching the slant which would encourage the flow to remain attached. A similar effect may have been obtained from specifying too low a level of dissipation rate or too high a level of turbulent kinetic energy at the inlet (the inlet turbulence levels were not specified in Han's paper). Furthermore, it may have been the simple log-law-based wall function employed by Han that was to blame for the overly large wall shear stress which, following Wilcox's analysis, was responsible for the base pressure being too low. At the 2002 ERCOFTAC workshop, Leschziner presented some preliminary results which indicated that the linear $k-\omega$ model of Wilcox was able to predict correctly the location and size of the separation bubble at the leading edge of the slant for the 25° Ahmed body. This would indicate that a model which responds more accurately to adverse pressure gradients is able to account for the complex behaviour on the rear of the Ahmed body.

Table 7.1 lists a number of other recent calculations of the Ahmed body which were submitted to the 2001 and 2002 ERCOFTAC Workshops on Refined Turbulence Modelling. In the final column of the table, a brief indication of the flow regime over the rear slant of the 25° Ahmed body is given. Only two simulations correctly predicted the appearance of a separation bubble at the leading edge of the 25° slant: the SST model results from Fluent[4] and the preliminary $k-\omega$ model results of Leschziner. The Fluent results are, however, somewhat in doubt as two other simulations of the same flow using the same SST model (from EC Nantes and CFX) predicted fully separated flow over the rear slant. This apparent conflict in results may be due to differences in the grid, boundary conditions and convection schemes employed.

[4]Fluent did not state explicitly that they used a low-Re modelling approach although they did use 2.3 million cells for the half-body (a mixture of tetrahedral and hexahedral cells). This compares to the calculations presented in this thesis which used $331,000$ cells for the half-body using wall functions.

Authors	**Turbulence Model**	**Slant Flow Mode**
Guilmineau & Queutey (EC Nantes)	low-Re SST $k-\omega$ model	fully separated
Braun, Lanfrit & Cokljat (Fluent)	realizable linear $k-\varepsilon$ SST $k-\omega$ model	fully attached separation bubble
Craft, Gant, Iacovides, Launder & Robinson (UMIST)	linear $k-\varepsilon$ with SCL wall function linear $k-\varepsilon$ with UMIST-N wall function realizable linear $k-\varepsilon$ with SCL WF realizable linear $k-\varepsilon$ with UMIST-A WF cubic non-linear $k-\varepsilon$ with SCL WF cubic non-linear $k-\varepsilon$ with UMIST-A WF	fully attached fully attached fully attached fully attached fully separated fully separated
Durand, Kuntz, Menter (EC Nantes/CFX)	linear $k-\varepsilon$ with scalable wall function low-Re SST $k-\omega$ model SSG DSM	fully attached fully separated fully attached
Leonard, Hirsch, Kovalev, Elsden, Hillewaert & Patel (Vrije Universiteit, Brussels/NUMECA)	low-Re linear $k-\varepsilon$ (Yang-Shih)	fully attached
Leschziner (Imperial College)	low-Re linear $k-\omega$ model	separation bubble

Table 7.1: RANS simulations of the 25° Ahmed body flow contributed to the 2001 and 2002 ERCOFTAC Workshops on Refined Turbulence Modelling.

There is insufficient information provided in the description of the computational methods to come to any firm conclusions. There is agreement amongst the numerous simulations which used a linear $k-\varepsilon$ model that the boundary layer remains fully attached for the entire length of the 25° slant. The actual velocity profiles over the slant differ somewhat between different linear $k-\varepsilon$ model calculations which can be attributed to slight differences in the models (realizable, low-Re or with wall functions), grid, boundary conditions and convection schemes. The realizable linear $k-\varepsilon$ model results from UMIST, in particular, showed a tendency towards a separated boundary layer at the downstream edge of the rear slant. Both the UMIST non-linear $k-\varepsilon$ model and the EC-Nantes/CFX SST model predicted fully separated boundary layers over the entire rear slant. Interestingly, both groups also found that running the same calculations again with a lower-order upwind convection scheme caused the flow over the rear slant to become fully attached. This remarkably strong effect of the convection scheme was analyzed in detail by Robinson [35] who concluded that the more diffuse side-edge vortex predicted using the lower-order scheme was able to draw a sufficient amount of fluid out of the boundary-layer on the slant to cause the flow to remain attached. Using a less-diffusive, higher-order convection scheme the side-edge vortex predicted by the non-linear model used by Robinson was insufficiently strong to cause the boundary layer to remain attached.

Hinterberger & Rodi submitted LES results for both the 25° and 35° Ahmed bodies to the 2001 ERCOFTAC workshop using a grid with approximately 8.8 million nodes and, the following year, submitted results using a refined grid of 18.5 million nodes. The results from these huge calculations (the more recent ones taking 30,000 CPU hours) showed no clear improvement in flow prediction compared to the RANS approaches. The time-averaged boundary-layer over the 25° slant was predicted as predominantly separated, in poor agreement with the experiments, although the Reynolds stress profiles over the rear slant showed better agreement with the experiments than any of the RANS models (indicating that the LES may have captured correctly the level of unsteadiness seen in the experiments). These slightly disappointing results may have been due to an averaging time used to generate the mean-flow data which was relatively low (4.5 non-dimensional time-steps[5]).

The above discussion has concentrated on the prediction of flow in the vicinity of the rear slant as this has the greatest contribution to the overall drag experienced by the Ahmed body. One could also examine the flow in the wake downstream from the body although experience from the workshops has shown that accurate prediction of the trailing vortices in the wake is strongly dependent upon predicting the correct mode of flow over the slant.

Suga *et al.* [141] studied a related flow, around the car body of Maeda *et al.* [142], using the low-*Re* Launder-Sharma $k-\varepsilon$ model and the three-equation cubic non-linear $k-\varepsilon-A_2$ model of Craft *et al.* [134]. The Maeda car body features a more rounded front section than the Ahmed body and a rear slant which extends from the top to the bottom surfaces (i.e. the vertical "base" surface which occurs on the rear of the Ahmed body is absent). The pressure coefficient distribution predicted by the $k-\varepsilon-A_2$ model was generally in good agreement with the experimental measurements, although a separation bubble was predicted near the leading edge of the 25° rear slant whereas the experiments suggested that there was almost no separation. The drag coefficient predicted by the linear $k-\varepsilon$ model was too high for all slant angles (25° to 55°) which was attributed to excessive turbulence levels generated around the leading edge of the body. The $k-\varepsilon-A_2$ model predicted more reasonable overall levels of drag coefficient, although the predictions were slightly too high at 25° (due to the separation bubble) and too low at 35° and 55°.

7.3 Computational Details

7.3.1 Models Used

The results from two calculations of the flow around the 25° Ahmed body are documented in this thesis. The first calculation used the standard linear $k-\varepsilon$ model of Launder & Spalding [143] in conjunction with the Simplified Chieng & Launder (SCL) wall function, the second used the same turbulence model with the low-*Re* damping functions of Launder & Sharma [13] and the new numerical wall function (UMIST-N). Both calculations employed the standard Yap correction. The UMIST-N wall function employed 45 subgrid nodes across the near-wall

[5]where one time-step is equivalent to the time taken for a packet of fluid to travel the length of the flow domain, from inlet to outlet.

cell, clustered near the wall using an expansion ratio of 1.09. This gave a maximum $y^+ < 0.1$ for the wall-adjacent subgrid node with a minimum of 10 subgrid nodes within $y^+ \approx 13$ (where $y^+ = c_\mu^{1/4} k^{1/2} y/\nu$). Increasing the number of subgrid nodes to 60 had no visible effect on the results of the simulation. It was originally intended that results would be obtained using the non-linear Craft *et al.* model [30] with the UMIST-N wall function, to complement the earlier work of Robinson (who used the Craft *et al.* model with two other wall functions). However, the combination of non-linear model and UMIST-N wall function was found to cause numerical instability in the calculations. Some suggestions for the possible causes of the stability problems are discussed in Section 8.3. Further investigation and correction of the stability problems was not possible within the framework of this thesis due to time and funding limitations.

7.3.2 Numerical Methods

The STREAM code, described in Section 3.3, was used to study the Ahmed body flow. Convection of both momentum and turbulence parameters was approximated using the UMIST differencing scheme discussed in Section 3.3.3. The flow was considered as steady and the symmetry plane was used so that a grid was constructed only around half of the body. Calculations were converged until residuals of the momentum, mass and turbulence equations were all below 5×10^{-6}, approximately an order-of-magnitude lower than that achieved in the previous calculations by Robinson [35] for the Ahmed body flow, which were considered to be fully converged. Under-relaxation factors are shown in Table 7.2.

U	V	W	P	k	ε
0.2	0.2	0.2	0.3	0.1	0.1

Table 7.2: Under-relaxation factors used for the Ahmed body flow.

7.3.3 Domain and Grid

Figures 7.7, 7.8 and 7.9 show the computational grid used to study the flow around the 25° Ahmed body. The grid was identical to that used in a previous study of the Ahmed body by Robinson [35] and employed 22 blocks (shown schematically in Figure 7.10) and approximately 331,000 cells. The legs, or stilts, on which the model was supported in the wind tunnel experiments, were not modelled in the computational grid. The effects of ignoring the stilts on the predicted drag and the flow field are discussed later. The near-wall distribution of cells was arranged to maintain main-grid y^+ values of as many as possible near-wall cells around the body to within the limits $30 < y^+ < 300$ (where $y^+ = c_\mu^{1/4} k^{1/2} y/\nu$), but these limits were exceeded in some regions of stagnation and boundary-layer separation or reattachment.

The y^+ values of the near-wall cells adjacent to the ground plane were not controlled since to maintain $y^+ < 300$ would have required high-aspect ratio cells which would have compromised the stability of the calculation. Due to the large number of nodes required to model the Ahmed body, it was not possible to refine the grids and establish grid independence. However, Robinson did examine a coarser grid, with 158,000 cells, to provide some information regarding grid independence. Using a realizable linear $k-\varepsilon$ model, the flow over the entire 25° rear slant was predicted to be fully attached with the coarse grid. In comparison, flow predictions using the finer (331,000 cell) grid showed a greater tendency towards separation of the boundary layer at the trailing edge of the slant. It was suggested that the enhanced "artificial diffusion" induced by the coarse grid helped to maintain attached flow over the rear slant.

7.3.4 Boundary Conditions

The computational domain used to study the Ahmed body flow is shown in Figure 7.11. The Simplified Chieng & Launder (SCL) wall function was always used for the floor boundary whereas around the surface of the Ahmed body either the SCL or the UMIST-N wall function were applied. The domain boundary along the centreline of the body (at $y = 0$), the opposite boundary at the outside limit of the domain ($y = 1.044$m) and the upper domain boundary ($z = 1.044$m) were all treated as symmetry planes. Ideally the upper and side domain boundaries would be treated with entrainment conditions. However, symmetry planes were used instead to provide a more stable calculation and were justifiable as there is little deflection of the flow at these boundaries. The downstream outlet was set with zero-gradient for all variables. Flat profiles (i.e. constant values) of velocity and turbulence parameters were set at the inlet plane, one body-length upstream of the Ahmed body. Values of the inlet streamwise velocity and turbulent kinetic energy were calculated by integrating the U- and k-profiles measured by Lienhart *et al.* across the flow domain at $x = -1.444$m. This resulted in an inlet bulk U-velocity which was lower (38.51ms^{-1}) than the stated experimental bulk U-velocity of 40ms^{-1}. The lower inlet bulk velocity corresponded to an inlet Reynolds number of $Re = 7.57 \times 10^5$ (based on the body's height) which compares to the original Ahmed experimental value of $Re = 1.18 \times 10^6$. The Reynolds number is sufficiently high that this minor adjustment should not have significantly influenced the results. The average inlet turbulent kinetic energy was calculated as $k_{in} = 6.58 \times 10^{-3}$m^{2}s^{-2} and the inlet dissipation rate was calculated from a viscosity ratio of $\nu_t/\nu = 10$. This viscosity ratio was recommended by Lienhart *et al.* following experimental measurements which estimated the Taylor microscale upstream of the body as $\lambda = 2.6$mm, where the dissipation rate is given by:

$$\varepsilon = \frac{2k}{\lambda^2}\nu \qquad (7.1)$$

7.4 Calculated Flow Results

Figures 7.13 and 7.14 show profiles of the streamwise mean U-velocity and RMS u-velocity at six measurement locations on the top surface of central portion of the Ahmed body, approximately midway between the nose and the slant (positions shown in Figure 7.12). There is practically no difference in the results obtained using the SCL and the UMIST-N wall functions. The predicted boundary layer thickness is slightly greater than that found in the experiments and the freestream velocity is also slightly higher in the calculations due to the blockage effect of the body and the use of symmetry conditions at the far-field boundaries (the body blocked approximately 5% of the cross-sectional area of the calculation flow domain).

If one examines more closely the near-wall region one can discern differences between the two wall function predictions. Figure 7.15 shows profiles of the mean U-velocity, turbulent kinetic energy and eddy-viscosity on the top surface of the Ahmed body (on the centreline) at $x = -831$mm, just downstream of the curved section near the nose. These profiles show nodal values (not interpolated values) and the UMIST-N wall function results show the subgrid distribution of U, k and μ_t across the near-wall cell. There are clear differences between the UMIST-N and SCL wall function results at this position, notably in the k and μ_t profiles. This can be attributed to the overprediction of k at a stagnation point associated with the linear $k - \varepsilon$ model being more pronounced with a low-Re type treatment, as is effectively provided by the UMIST-N wall function, than with standard wall functions (as discussed in Chapter 5). In addition, the UMIST-N wall function accounts for the effects of pressure gradient and convection which are neglected by the SCL wall function. As one moves further downstream along the top of the Ahmed body to a region where the flow is closer to equilibrium, the differences between the two wall function treatments becomes less pronounced (see Figure 7.16). Progressing still further along the top (Figures 7.17 and 7.18) one can see differences in the two wall function predictions becoming apparent again (although these differences are less than are observed in Figure 7.15). Towards the leading edge of the slant, situated at $x = -201.2$, the streamwise velocity increases and k decreases everywhere except very near the wall. This trend for k decreasing is what one would expect in an accelerating boundary-layer flow [12]. On the slant surface (Figures 7.19, 7.20 and 7.21) a more pronounced difference in the levels of U, k and μ_t arises between the two wall function predictions although the profiles are of similar general shape.

Figures 7.22 – 7.27 show centreline profiles of the mean-velocity, Reynolds stresses and turbulent kinetic energy over the rear slant. Due to the scales needed in order to fit the multiple profiles into one figure, the differences between the two wall function predictions are barely discernible although differences between the predicted and experimental results can be seen quite clearly. These differences are particularly visible for the Reynolds stress profiles where the experimental values are nearly an order-of-magnitude larger than those predicted by the two wall functions. The significant underprediction of the Reynolds stresses and turbulent kinetic energy was a feature of all the RANS simulations of the Ahmed body which were submitted to the two ERCOFTAC workshops. Only the LES results submitted to the workshops came close to predicting the experimental Reynolds stress profiles.

The U-velocity profiles, Figure 7.22, show that the mean-flow recirculation bubble, observed in the experiments, existed from the leading edge of the slant to around $x = -100$mm. With the linear $k - \varepsilon$ model, both wall functions predicted a fully attached boundary layer for the whole length of the slant. This can be seen more clearly in Figure 7.28 which shows velocity vectors and pressure coefficient contours over the rear slant as viewed from above[6]. The two wall function predictions are similar with the UMIST-N wall function showing a minor increase in velocity near the downstream edge of the slant as compared to the SCL wall function.

Figure 7.29 compares the resultant velocity vector at the near-wall main-grid node to that at the wall-adjacent subgrid node over the rear slant of the Ahmed body, using the UMIST-N wall function. There is considerable skewing of the velocity vector across the subgrid. Near the side edge of the rear slant the subgrid and main-grid vectors are misaligned by up to 90°. The subgrid results also show that there was a reattachment line parallel to the side edge of the slant surface, similar to that shown in the experimental results of Ahmed *et al.* (see Figure 7.3) and Spohn & Gilliéron (marked "B" in Figure 7.6). Skewing of the velocity profile across the near-wall main-grid cell is ignored by standard log-law-based wall functions (such as the SCL treatment) which assume that the wall-shear stress is aligned to the resultant velocity vector at the near-wall main-grid node position.

Figures 7.30, 7.31 and 7.32 chart the growth of the wake from near the leading edge of the slant to a position approximately one-half of a body length downstream (from $x = -178$mm to $x = 500$mm). Experimental measurements of turbulent kinetic energy and mean velocity are compared to those predicted by the linear $k-\varepsilon$ model with UMIST-N wall function (those obtained with the linear $k - \varepsilon$ model and SCL wall function are similar and are not shown). The underprediction of k near the slant surface, discussed above, is shown clearly on Figures 7.30 and 7.31. In addition, the measured turbulent kinetic energy is significantly higher than that predicted around the bottom of the model, near the wind-tunnel wall. This latter feature is mainly due to the enhanced turbulence levels generated by the stilts on which the body was supported in the wind tunnel, which were neglected in the computational model. Over the rear slant the predicted side-edge vortex has the same location but is weaker than the measured vortex. Further downstream, the predicted and measured trailing vortices are in remarkably good agreement both in terms of position and strength (Figure 7.32). However, contours of turbulent kinetic energy continue to show poor agreement in the wake downstream of the body with measured peak values approximately twice the peak predicted values.

In order to provide a brief comparison of the current computations with those of Robinson

[6]The resultant velocity vectors are obtained from the main-grid node values with the UMIST-N wall function, not the subgrid values.

[35] (who used the same code and an identical grid but with different turbulence models) Figures 7.33 and 7.34 show contours of the turbulent kinetic energy at the centreline plane around the Ahmed body. As previously discussed, there are some minor differences between the current linear $k-\varepsilon$ model predictions using the UMIST-N and SCL wall functions (Figure 7.33). The linear $k-\varepsilon$ model predictions of Robinson using the realizability constraint of May [144], shown in Figure 7.34, have slightly lower levels of k in the impingement region on the nose of the Ahmed body and higher values of k over the slant and in the wake, although the flow over the slant is still fully attached. The Craft *et al.* [30] non-linear $k-\varepsilon$ model also predicted low turbulence levels on the nose of the Ahmed body and a fully separated boundary layer over the rear slant with a larger wake.

Drag Predictions

Tables 7.3 and 7.4 compare drag coefficients for the Ahmed body predicted by the present computations to those of Robinson [35] and the experimental measurements of Ahmed *et al.* [137] and Lienhart *et al.* [138]. The reference pressure used to calculate the pressure coefficient $\left(c_p = \left(P - P_{ref}\right)/0.5\rho U_{bulk}^2\right)$ in both sets of computational results was taken at $(x, y, z) = (-977\text{mm}, 0\text{mm}, 912\text{mm})$ – vertically above the nose of the body on the centreline plane. The values shown from Ahmed *et al.* have been extracted from Figure 7.2 which plots the breakdown of the drag coefficient for slant angles from 0° to 40° . It should be noted that Ahmed *et al.* only actually measured the components of drag at four slant angle 5°, 12.5°, 30° (high drag) and 30° (low drag). These measurements were then interpolated to produce the continuous distribution shown in Figure 7.2. In addition, Ahmed *et al.* calculated the friction drag by subtracting the three pressure-drag components (on the nose, slant and base) from the total drag force on the car body. Any errors in the pressure-drag measurements would therefore effect the value of the friction drag. There are differences between the pressure coefficients extracted from Ahmed *et al.*'s graph and the more recent Lienhart *et al.* measurements. The Lienhart *et al.* measurements show the pressure coefficient on the slant to be only around 36% higher than that on the base, compared to Ahmed *et al.*'s assumed difference of 100%. In addition, Lienhart *et al.*'s values are slightly higher overall than those of Ahmed *et al.* It is not possible to identify whether the differences between the two sets of experimental data are due to differences in the location of the reference pressure[7], inaccurate interpolation or experimental error since Lienhart *et al.* did not measure the total drag or the pressure coefficient on the nose.

The total drag coefficient predicted by the current calculations using the linear $k-\varepsilon$ model are close to those obtained previously by Robinson with a realizable linear $k-\varepsilon$ model. There are only slight differences between the linear $k-\varepsilon$ model predictions with the SCL and UMIST-N wall function values (9.1% error for the SCL wall function compared to 8.4% with

[7] The location of the reference pressure was not specified by Ahmed *et al.*

UMIST-N). The ratio of pressure coefficient on the slant to that on the base is slightly higher with the linear model than the realizable model although it is difficult to make conclusions about the accuracy of the two models due the discrepancies in the experimental measurements mentioned above.

Table 7.3 shows that Robinson's calculations using the non-linear model with the SCL wall function gave the total drag coefficient in best agreement with Ahmed *et al.*'s measurements. However, it would be misleading to conclude from this that the model provided the best predictions of the flow since it also predicted a fully-separated boundary layer over the rear slant, whereas in fact the flow was predominantly attached. The difference in the predicted mode of the flow over the rear slant is indicated by the balance between the slant and base pressure coefficients, shown in Table 7.4. The non-linear model predicted the pressure coefficient on the base to be greater than that on the slant whilst the linear models (and the experiments) predicted the opposite. This shows the importance of looking at the breakdown of drag and not just the total value.

Case	**No Stilt**	**With Stilt**	**% Error**
Ahmed *et al.* [137]	-	0.285	-
linear $k-\varepsilon$, SCL	0.294	0.311	**9.1**
linear $k-\varepsilon$, UMIST-N	0.292	0.309	**8.4**
realizable $k-\varepsilon$, SCL (*)	0.294	0.311	**9.1**
realizable $k-\varepsilon$, UMIST-A (*)	0.293	0.310	**8.8**
non-linear $k-\varepsilon$, SCL (*)	0.267	0.286	**0.4**
non-linear $k-\varepsilon$, UMIST-A (*)	0.251	0.270	**-5.3**

Table 7.3: Total drag for the 25° Ahmed body. Realizable and non-linear $k-\varepsilon$ model results, marked with an asterisk (*), are taken from Robinson [35]. Allowance for the drag due to the stilts (on which the body is supported in the wind-tunnel) is based on the drag on a circular cylinder (see [35]).

Case	**Pressure Coefficient**			**Skin**
	Nose	**Slant**	**Base**	**Friction**
Ahmed *et al.* [137]	0.020	0.140	0.070	0.055
Lienhart *et al*. [138]	-	0.158	0.116	-
linear $k-\varepsilon$, SCL	0.055	0.140	0.095	0.004
linear $k-\varepsilon$, UMIST-N	0.055	0.141	0.092	0.004
realizable $k-\varepsilon$, SCL (*)	0.048	0.139	0.103	0.004
realizable $k-\varepsilon$, UMIST-A (*)	0.049	0.139	0.100	0.005
non-linear $k-\varepsilon$, SCL (*)	0.047	0.105	0.111	0.004
non-linear $k-\varepsilon$, UMIST-A (*)	0.051	0.083	0.114	0.004

Table 7.4: Drag breakdown for the 25° Ahmed body. Realizable and non-linear $k-\varepsilon$ model results, marked with an asterisk (*), are taken from Robinson [35].

Influence of $\partial P/\partial \zeta$ in the UMIST-N Wall Function

In Chapter 4 it was noted that the pressure-gradient term appearing in the non-orthogonal UMIST-N wall function momentum equations involves gradients of pressure in all three co-ordinate directions. For example, the pressure-gradient term for the subgrid U-momentum equation is given by:

$$\sqrt{g_{11}}\left(g^{11}\frac{\partial P}{\partial \xi}+g^{12}\frac{\partial P}{\partial \eta}+g^{13}\frac{\partial P}{\partial \zeta}\right) \tag{7.2}$$

where ξ and η are the two wall-parallel coordinates, ζ is not parallel to the wall (but is not necessarily wall-normal) and g^{ij} is the contravariant metric tensor. The pressure distribution across the near-wall cell is determined by solving the following equation:

$$\nabla P\cdot\hat{\mathbf{n}}+(\nabla\cdot\rho\overline{\mathbf{u}\otimes\mathbf{u}})\cdot\hat{\mathbf{n}}=0 \tag{7.3}$$

where $\hat{\mathbf{n}}$ is the unit wall-normal vector, $\overline{\mathbf{u}\otimes\mathbf{u}}$ is the Reynolds stress in vector form. It is recognized that Equation (7.3) only provides an approximation of $\partial P/\partial \zeta$. To resolve accurately the pressure distribution in the near-wall region would entail the solution of a pressure-correction equation across the subgrid domain which would drastically increase the computational time (in effect one would be performing a full low-Reynolds-number model simulation).

To examine whether $\partial P/\partial \zeta$ is a significant term in the subgrid momentum equations and whether the above approximation is satisfactory, profiles have been plotted of the near-wall velocity in a region of the Ahmed body flow where the grid is highly skewed. Figure 7.35 shows the wall-parallel velocity in five locations around the 90° rear corner of the Ahmed body on the centreline (shown in Figure 7.36). The grid in this region has skewed cells of angle 45°. One can see that the velocity on the underside of the Ahmed body (profiles A and B on the left of Figure 7.35) increases smoothly from zero on the wall surface up to a maximum of around 0.8 (where 1.0 is the free-stream velocity). The velocity in profile C, just around the corner on the base surface, is lower near the wall and further up the base surface (profiles D and E) the wall-parallel velocity changes sign. Figure 7.37 shows the distribution of three subgrid pressure-gradient terms: $\left(\sqrt{g_{11}}g^{11}\partial P/\partial\xi\right)$, $\left(\sqrt{g_{11}}g^{12}\partial P/\partial\eta\right)$ and $\left(\sqrt{g_{11}}g^{13}\partial P/\partial\zeta\right)$, for the corner cell C (for this cell the ξ-direction is aligned to the main Ahmed body z-axis, the η-direction is aligned to the y-axis and the ζ-direction at angle of 45° to the base surface, as shown in Figure 7.36). The two wall-parallel components, involving $\partial P/\partial\xi$ and $\partial P/\partial\eta$, are assumed constant across the subgrid and, as would be expected, the ξ-direction component is far greater than the transverse η-direction component. The calculated $\left(\sqrt{g_{11}}g^{13}\partial P/\partial\zeta\right)$ profile is non-uniform, having a maximum near the outer edge of the subgrid domain and decreasing to a quarter of that value near the wall. Near the outer edge of the cell, the term involving $\partial P/\partial\zeta$ is nearly three times as large as the $\partial P/\partial\xi$ term.

If the $\partial P/\partial\zeta$ term is removed from the Ahmed body calculation (by simply setting the term to zero) and the calculation run on from a pre-converged state for 500 iterations, the velocity profiles shown in Figure 7.38 are obtained. The principal effect of switching off $\partial P/\partial\zeta$ is in profile C where there is a marked increase in the subgrid velocity leading to a large strain-rate at the outer edge of the subgrid. Interestingly, the main-grid velocity field is practically unchanged by switching off the subgrid $\partial P/\partial\zeta$ even though the wall shear stress calculated from the subgrid solution is increased by approximately 86%. Plots of the overall main-grid flow behaviour, such as those shown in Figures 7.28 and 7.33, are also indistinguishable, meaning that the main-grid domain is little affected by the $\partial P/\partial\zeta$ term in the subgrid. The reason that the calculation was run for only 500 iterations, rather than until a fully-converged solution was reached, was that the calculation became unstable when the $\partial P/\partial\zeta$ term was removed from the subgrid momentum equations[8]. This instability was likely to have been due to the large strain-rate observed at the outer edge of the subgrid domain which caused large production and dissipation source terms in the subgrid k and $\tilde{\varepsilon}$ equations which, in turn, affected the main-grid solution.

To summarize the above discussion, the subgrid $\partial P/\partial\zeta$ term has been shown to have a significant effect on the subgrid velocity profile in a region of the flow where the cells are highly skewed and there are large pressure gradients. In the particular region of the Ahmed flow which has been examined, the calculated subgrid velocity profile has little effect on the near-wall main-grid velocity. This would indicate that the approximation of the subgrid $\partial P/\partial\zeta$ has little influence upon the overall flow field, although an inaccurate estimation of the subgrid pressure distribution may lead to large subgrid strain-rates which compromise the stability of the calculation.

7.5 Discussion & Conclusions

It has been shown that it is possible to apply the UMIST-N wall function to a complex three-dimensional flow involving a non-orthogonal multiblock grid arrangement. The prediction of the 25° Ahmed body flow using the new wall function was shown to be similar to that obtained using a standard log-law based wall function. This is surprising since the new wall function did predict skewing of the velocity vector across the near-wall cell, which was anticipated to be influential in the formation of the side-edge vortices over the rear slant. Although the results are therefore slightly disappointing the linear $k-\varepsilon$ model predictions with the UMIST-N wall function are broadly in agreement with the low-Re linear $k-\varepsilon$ model predictions of Leonard *et al.*[9] (both approaches predicted a fully-attached boundary layer over the rear slant). It

[8]Momentum and turbulent kinetic energy residuals oscillated at a level around order-of-magnitude greater than that previously achieved when the $\partial P/\partial\zeta$ term was included. The dissipation-rate residuals also increased by approximately four orders-of-magnitude. Analysis of the velocity and turbulence parameters in the regions of maximum residual (located near the 90° rear corner of the Ahmed body) showed small changes in the profiles between successive iterations.

[9]The results of Leonard *et al.* were presented at the 2002 ERCOFTAC Workshop on Refined Turbulence Modelling (see Footnote 3 on Page 138).

appears, following the workshop comparisons, that the turbulence model used in the main-grid region of the flow domain has a greater influence on mode of the flow on the rear slant than the particular near-wall treatment employed. It would be interesting to examine this issue further, perhaps by making direct comparisons between a low-Re $k-\omega$ model (which predicted the separation bubble on the rear slant in almost perfect agreement with the experiments) and a $k-\omega$ model with different wall functions.

In the impinging-jet and spinning-disc flows considered in earlier chapters, the computing times of the UMIST-N wall function calculations were compared against the times for full low-Re model and standard log-law-based wall function calculations. A low-Re model simulation of the Ahmed body flow was not undertaken as part of the present study due to the computing requirements. Both the SCL and UMIST-N wall function calculations have been started from previous simulations, rather than from scratch, making it difficult to compare total CPU requirements. One can, however, compare computing times for the SCL and UMIST-N treatments on a per-iteration basis, as shown in Table 7.5. Ahmed body calculations with the UMIST-N wall function took around 24% greater time per iteration than those with the SCL wall function. This compares to an increase of around 60% for the impinging-jet flow and 100% for the spinning-disc flow.

Wall Function	CPU Time for 100 Iterations (s)	Relative CPU time
SCL	1022	1
UMIST-N	1268	1.24

Table 7.5: Computing times for 100 iterations of the Ahmed body calculation using the linear $k-\varepsilon$ model with the log-law-based Simplified Chieng & Launder (SCL) wall function and the new UMIST-N wall function. Computations were carried out on a 2.2GHz Pentium 4 processor with the same levels of compiler optimization.

One of the features of the new wall function which only becomes significant in a non-orthogonal grid arrangement, namely the appearance $\partial P/\partial\zeta$ in the subgrid momentum equations, has been shown to be an important term in the subgrid momentum equations in regions where the cells are highly skewed and there are large pressure gradients. The term was shown to have little or no effect on the overall main-grid solution although stability of the calculation was compromised if the term was neglected from the subgrid momentum equations. This indicates that the safest strategy is to use a grid which does not contain highly skewed subgrid cells.

In future simulations of the Ahmed body flow it would be interesting to examine the effect of using entrainment conditions on the top and side edges of the flow domain where, in the present calculations, symmetry conditions have been imposed. Menter noted at the 2002 ERCOFTAC workshop that specifying symmetry conditions instead of the more realistic entrainment conditions caused an increase in the velocity of the flow around the body due to the blockage effect which led to an increase in the predicted drag coefficient (an effect which was proportional to the square of the blockage ratio). The choice of boundary conditions was not however reported to affect the flow structure on the rear slant.